Communication Engineering

Ch.03. Amplitude Modulation

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Modulation Types (in earnest…)

- Continuous-wave (CW) modulation: *sinusoidal carrier* (amplitude, phase, frequency)
  - Ch3. Amplitude modulation: **AM**, **DSB-SC**, **SSB**, **VSB**
  - Ch4. Angle modulation: **PM**, **FM**
- Pulse modulation: *periodic pulse train* (amplitude, duration, positon): Ch5
  - Analogue pulse modulation: **PAM**, **PDM**, **PPM**
  - Digital pulse modulation: **PCM**, **DM**, **DPCM** → "0/1"
    ***Line code**: 0/1 → electrical representation
    =pulse shaping (Ch6) for "digital baseband modulation/transmission"
- Ch.6: Baseband transmission: discrete pulse-amplitude modulation → transmitted over a *low-pass channel* (e.g., a coaxial cable): **PAM**
- **Ch.7**: Digital band-pass (passband) modulation → transmitted over a *band-pass channel* (e.g., **wireless channel**): **ASK**, **PSK**, **FSK**, **QAM**
- **Ch. 8: Random signals and noise**
- **Ch. 9&10?**

3.0 What We Will Learn & Think ?

- Modulation
  - The process by which *some characteristic of a carrier wave* is varied in accordance with an *information-bearing signal*.
  - Continuous-(carrier) wave modulation
    - Amplitude modulation
    - Frequency modulation

$$c(t) = A_c \cos(2\pi f_c t) \quad (3.1)$$

- AM modulation family
  - Amplitude modulation (AM)
  - Double sideband-suppressed carrier (DSB-SC)
  - Single sideband (SSB)
  - Vestigial sideband (VSB)

3.0 What We Will Learn & Think ?

- Lesson 1 : Fourier analysis provides a powerful mathematical tool for developing mathematical as well as physical insight into the spectral characterization of linear modulation strategies
- Lesson 2 : The implementation of analog communication is significantly simplified by using AM, at the expense of transmitted power and channel bandwidth
- Lesson 3 : The utilization of transmitted power and channel bandwidth is improved through well-defined modifications of an amplitude-modulated wave's spectral content at the expense of increased system complexity.

Table 1: Summary of Amplitude Modulation Schemes (ED/CD: envelope/coherent detector).

Modulation Scheme	AM	DSB-SC	SSB	VSB	VSB+Carrier
Tx power cons (carrier and/or sid					carrier and side frequencies
Bandwidth (W, 2					$W + f_v$
Demodulation (ED or CD)	ED	CD			

You should find your feet and you will!

3.1 Amplitude Modulation

❑ Theory

- Message signal: modulating wave $m(t)$

- Sinusoidal carrier wave (cos not sin): modulated wave

$$c(t) = A_c \cos(2\pi f_c t) \quad (3.1)$$

where A_c : carrier amplitude
f_c : carrier frequency

- Amplitude-modulated wave

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t) \quad (3.2)$$

where k_a : *amplitude sensitivity* of the modulator

3.1 Amplitude Modulation

- The envelope of $s(t)$ has the same shape as the message signal $m(t)$ if two conditions are satisfied :
  - The amplitude of $k_a m(t)$ is always less than unity

    $$|k_a m(t)| < 1, \quad \text{for all } t \quad (3.3)$$

  - The carrier frequency f_c is much greater than the highest frequency component W (message bandwidth) of $m(t)$

    $$f_c >> W \quad (3.4)$$

- Envelope detector
  - A device whose output traces the envelope of the AM wave acting as the input signal
  - Demodulation of AM wave is achieved by using an envelope detector.

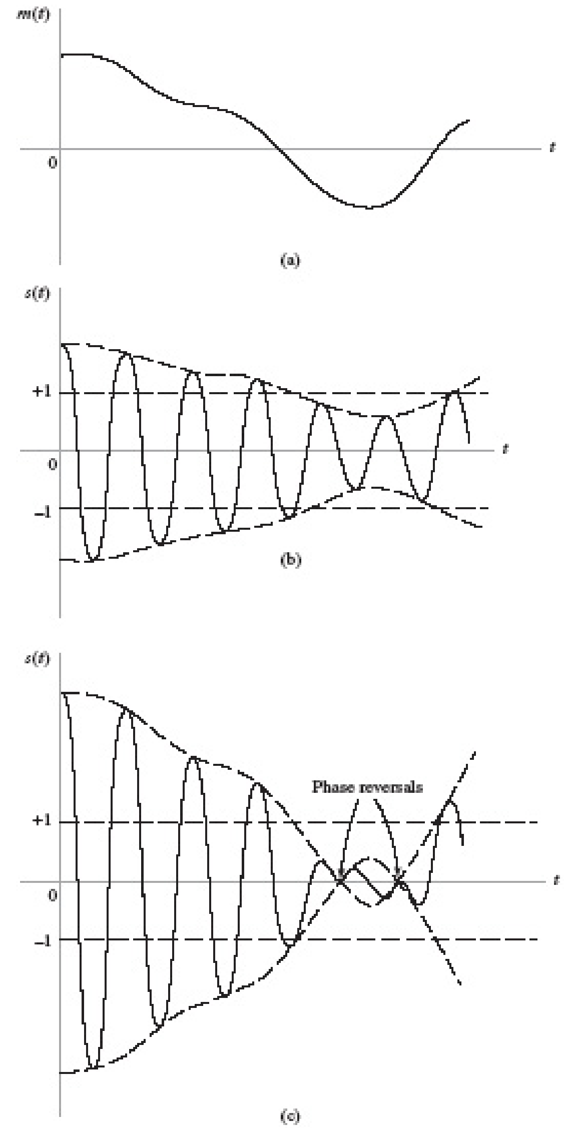


FIGURE 3.1 Illustration of the amplitude modulation process. (a) Message signal $m(t)$. (b) AM wave for $|k_a m(t)| < 1$ for all t. (c) AM wave for $|k_a m(t)| > 1$ for some t.

3.1 Amplitude Modulation

❑ Frequency-domain description of AM

- Message spectrum $M(f)$ = Fourier transform of $m(t)$
- The Fourier transform or spectrum of the AM wave $s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)] \quad (3.5)$$

where we used

$$\cos(2\pi f_c t) = \frac{1}{2}[\exp(j2\pi f_c t) + \exp(-j2\pi f_c t)]$$

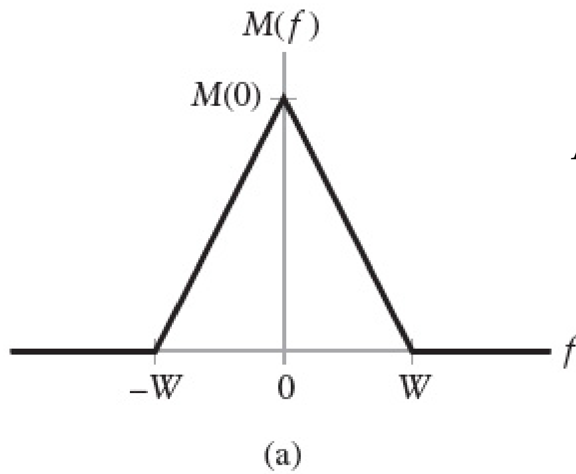
$$\exp(j2\pi f_c t) \Leftrightarrow \delta(f - f_c)$$

$$\cos(2\pi f_c t) \Leftrightarrow \frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

$$m(t)\exp(j2\pi f_c t) \Leftrightarrow M(f - f_c)$$

$$m(t)\cos(2\pi f_c t) \Leftrightarrow \frac{1}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

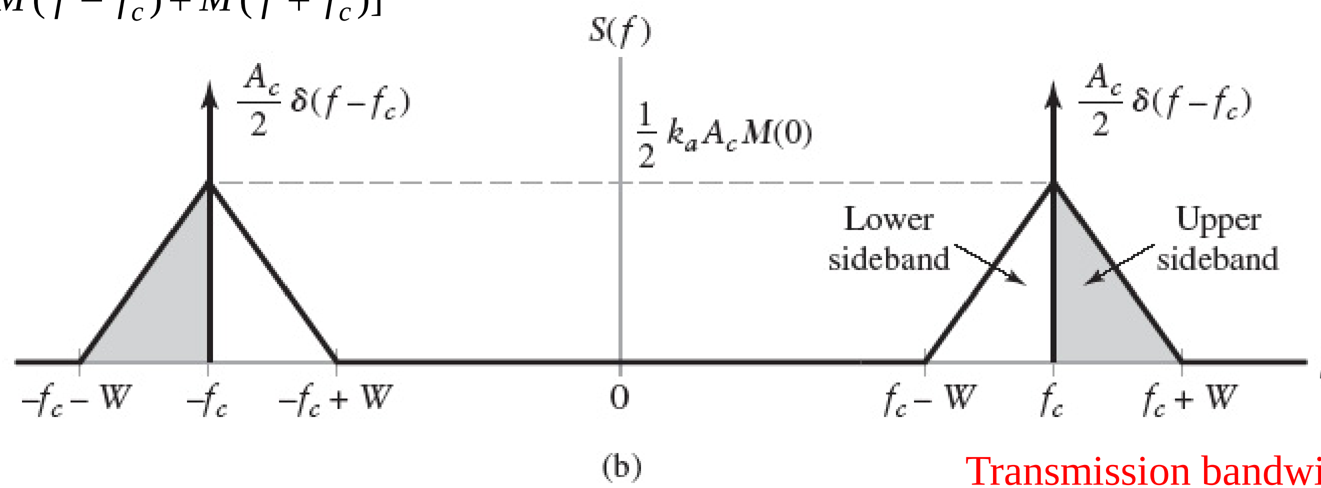
3.1 Amplitude Modulation



Assumption: $m(t)$ is band-limited.

Message bandwidth = W

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$



Transmission bandwidth = 2W

FIGURE 3.2 (*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of AM wave $s(t)$.

3.1 Amplitude Modulation

- Important observations from the spectrum of Fig. 3.2(b)

1. As a result of the modulation process, the spectrum of the message signal $m(t)$ for negative frequencies becomes visible for positive frequencies if $f_c > W$.
2. For positive frequencies, the portion of the spectrum of an AM wave lying above the carrier frequency f_c is referred to as the upper sideband, whereas the symmetric portion below f_c is referred to as the lower sideband.
3. The transmission bandwidth B_T of the AM wave;

$$B_T = 2W \quad (3.6)$$

3.1 Amplitude Modulation

EXAMPLE 3.1 Single-Tone Modulation

Consider a modulating wave $m(t)$ that consists of a single tone or frequency component; that is,

$$m(t) = A_m \cos(2\pi f_m t)$$

where A_m is the amplitude of the sinusoidal modulating wave and f_m is its frequency (see Fig. 3.3(a)). The sinusoidal carrier wave has amplitude A_c and frequency f_c (see Fig. 3.3(b)). The corresponding AM wave is therefore given by

$$s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t) \tag{3.7}$$

where

$$\mu = k_a A_m$$

The dimensionless constant μ is called the *modulation factor*, or the *percentage modulation* when it is expressed numerically as a percentage. To avoid envelope distortion due to overmodulation, the modulation factor μ must be kept below unity, as explained previously.

3.1 Amplitude Modulation

Figure 3.3(c) shows a sketch of $s(t)$ for μ less than unity. Let $A_{\max}$ and $A_{\min}$ denote the maximum and minimum values of the envelope of the modulated wave, respectively. Then, from Eq. (3.7) we get

$$\frac{A_{\max}}{A_{\min}} = \frac{A_c(1 + \mu)}{A_c(1 - \mu)}$$

Rearranging this equation, we may express the modulation factor as

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

Expressing the product of the two cosines in Eq. (3.7) as the sum of two sinusoidal waves, one having frequency $f_c + f_m$ and the other having frequency $f_c - f_m$, we get

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2}\mu A_c \cos[2\pi(f_c - f_m)t]$$

The Fourier transform of $s(t)$ is therefore

$$\begin{aligned} S(f) = {} & \frac{1}{2}A_c[\delta(f - f_c) + \delta(f + f_c)] \\ & + \frac{1}{4}\mu A_c[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)] \\ & + \frac{1}{4}\mu A_c[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)] \end{aligned}$$

Thus the spectrum of an AM wave, for the special case of sinusoidal modulation, consists of delta functions at $\pm f_c$, $f_c \pm f_m$, and $-f_c \pm f_m$, as shown in Fig. 3.3(c).

3.1 Amplitude Modulation

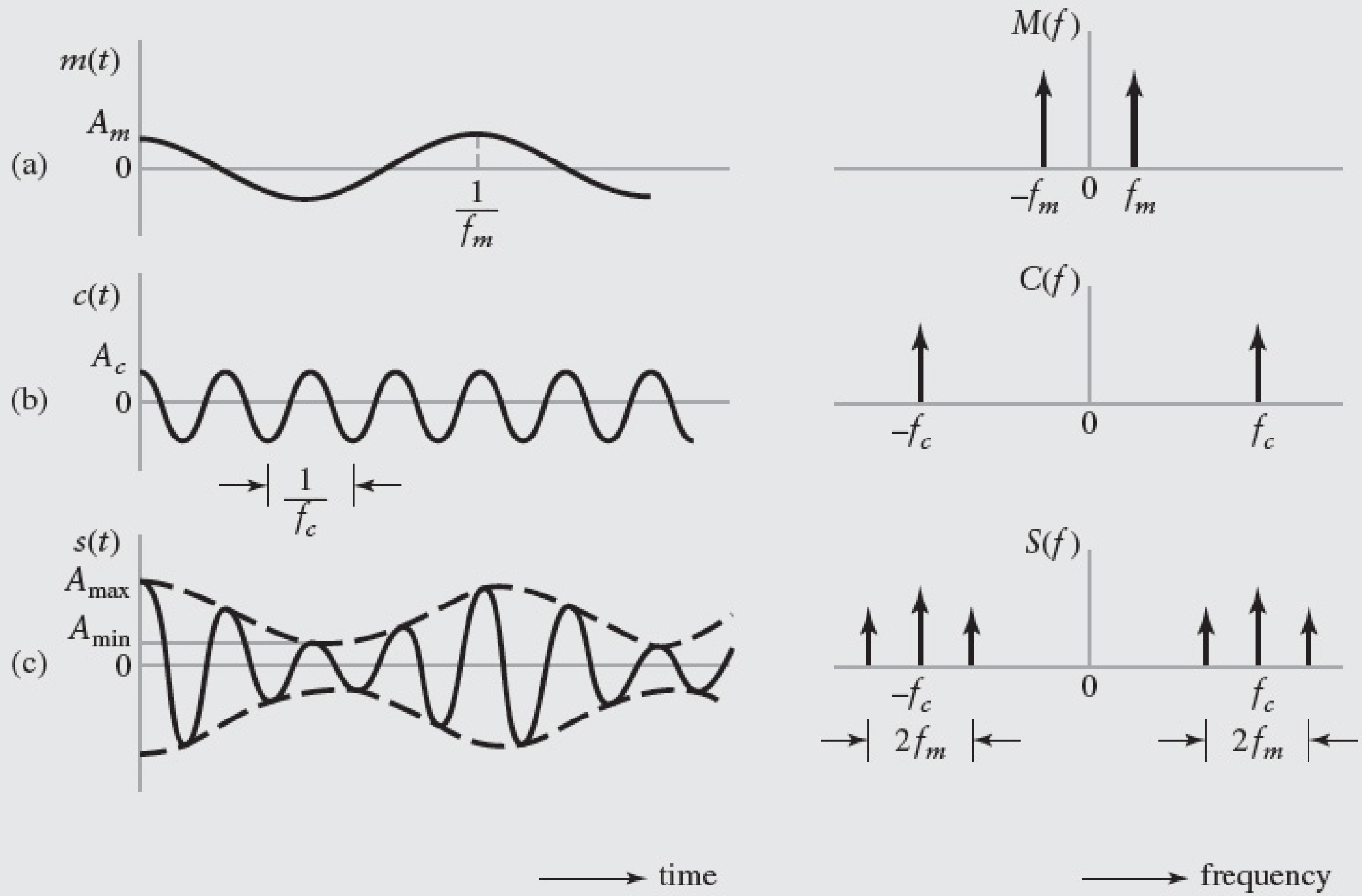


FIGURE 3.3 Illustration of the time-domain (on the left) and frequency-domain (on the right) characteristics of amplitude modulation produced by a single tone. (*a*) Modulating wave. (*b*) Carrier wave. (*c*) AM wave.

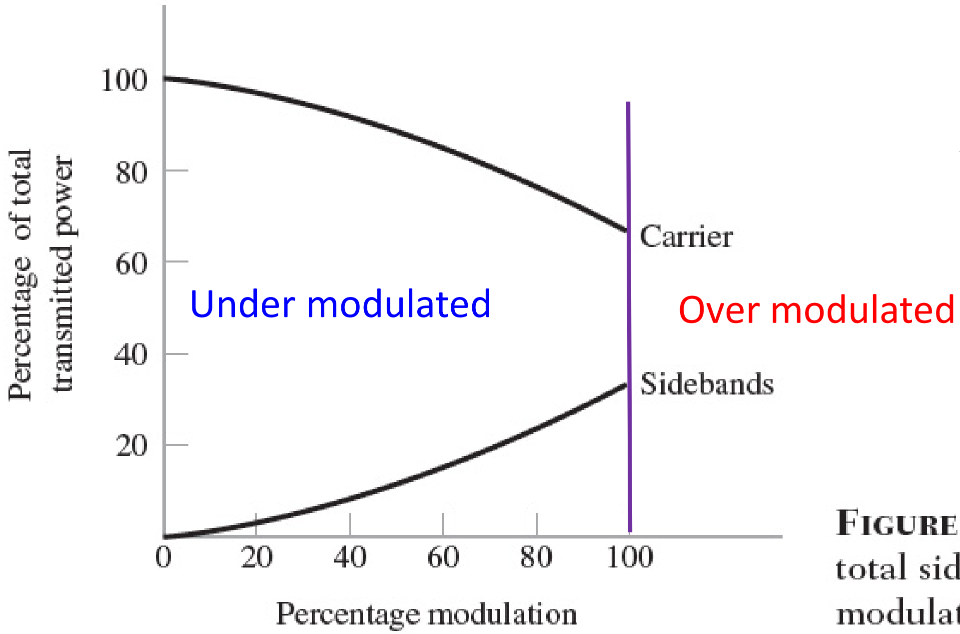
3.1 Amplitude Modulation

- The average power delivered to a 1-ohm resistor by s(t) is comprised of three components

$$\text{Carrier power} = \frac{1}{2} A_c^2$$

$$\text{Upper side} - \text{frequency power} = \frac{1}{8} \mu^2 A_c^2$$

$$\text{Lower side} - \text{frequency power} = \frac{1}{8} \mu^2 A_c^2$$



where μ : modulation factor or percentage modulation

FIGURE 3.4 Variations of carrier power and total sideband power with percentage modulation in amplitude modulation.

3.1 Amplitude Modulation

- Computer experiment : AM
  - We will study sinusoidal modulation based on the following parameters

$$\text{Carrier amplitude,} \quad A_c = 1$$
$$\text{Carrier frequency,} \quad f_c = 0.4Hz$$
$$\text{Modulation frequency,} \quad f_m = 0.05Hz$$

  - It is recommended that the number of frequency samples satisfies the condition

$$M \geq \frac{f_s}{f_r} = \frac{10}{0.005} = 2000$$

  - The modulation factor μ

$$\mu = 0.5, \text{corresponding to undermodulation}$$
$$\mu = 1.0, \text{corresponding to 100 percent modulation}$$
$$\mu = 2.0, \text{corresponding to overmodulation}$$

3.1 Amplitude Modulation

- Modulation factor $\mu=0.5$
  - The lower side frequency, the carrier, and the upper side frequency are located at $(f_c-f_m)=\pm0.35$ Hz, $f_c=\pm0.4$ Hz, and $(f_c+f_m)=\pm0.45$ Hz.
  - The amplitude of both side frequencies is $(\mu/2)=0.25$ times the amplitude of the carrier
- Modulation factor $\mu=1.0$
- Modulation factor $\mu=2.0$

3.1 Amplitude Modulation

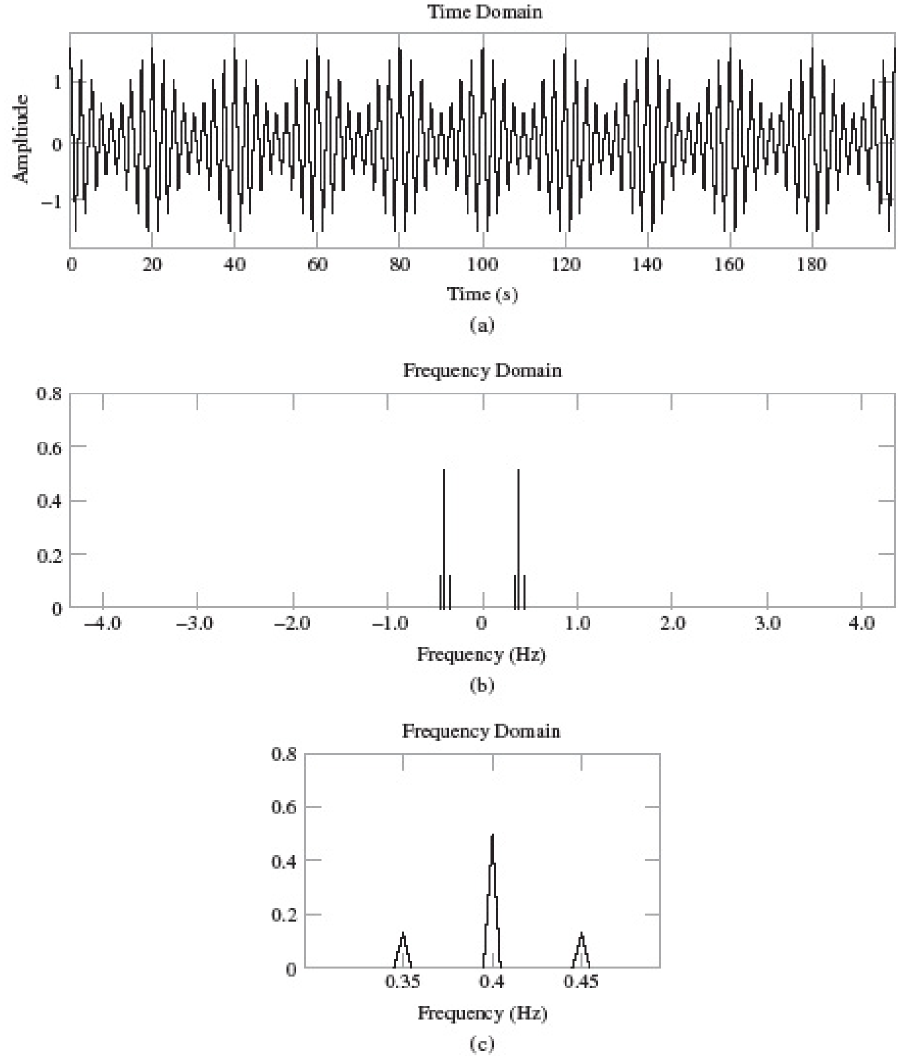


FIGURE 3.5 Amplitude modulation with 50 percent modulation: (*a*) AM wave, (*b*) magnitude spectrum of the AM wave, and (*c*) expanded spectrum around the carrier frequency.

Modulation factor $\mu=0.5$

3.1 Amplitude Modulation

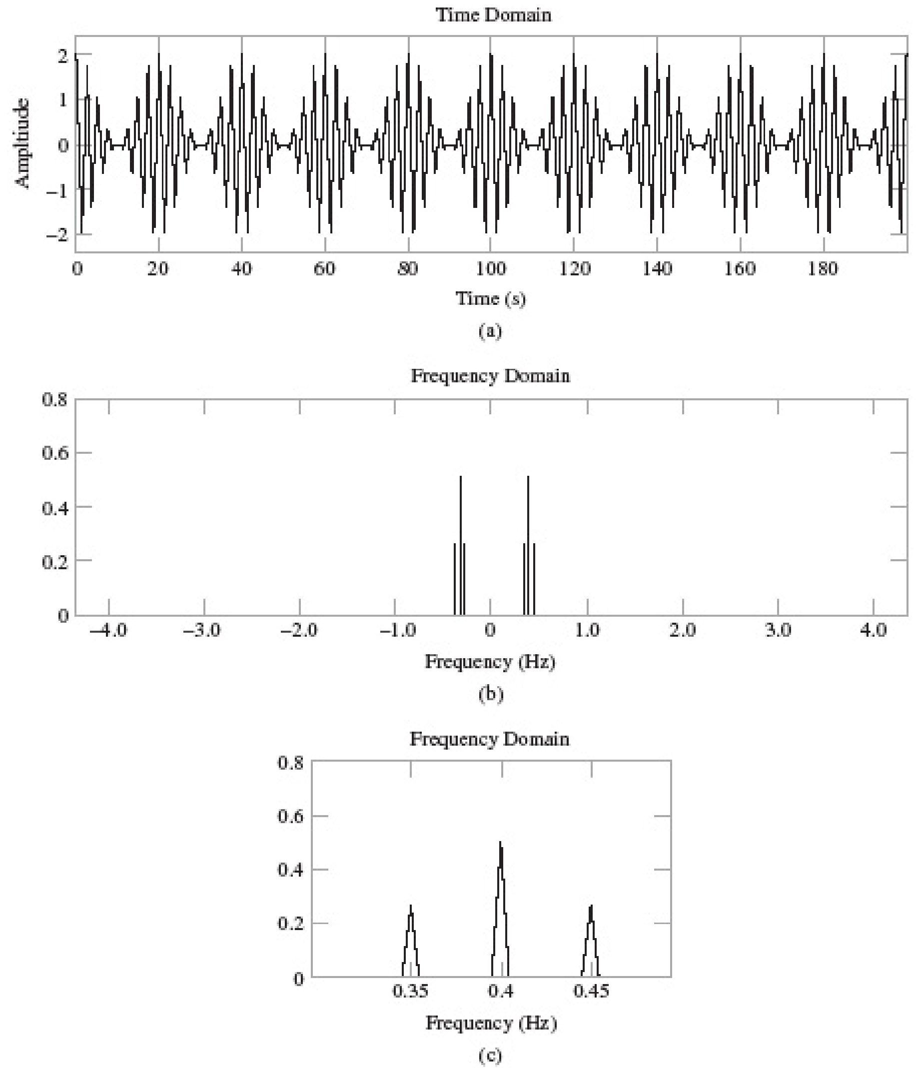


FIGURE 3.6 Amplitude modulation with 100 percent modulation: (*a*) AM wave, (*b*) magnitude spectrum of the AM wave, and (*c*) expanded spectrum around the carrier frequency.

Modulation factor μ=1.0

3.1 Amplitude Modulation

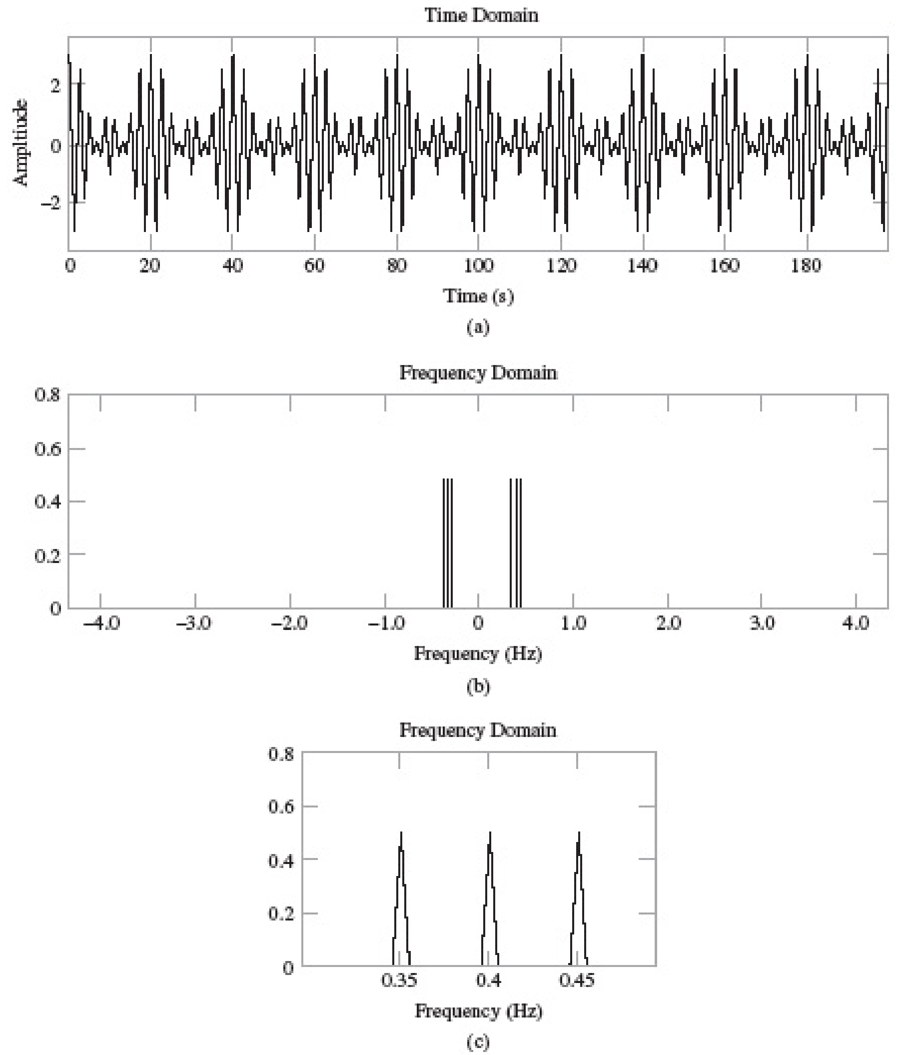


FIGURE 3.7 Amplitude modulation with 200 percent modulation: (*a*) AM wave, (*b*) magnitude spectrum of the AM wave, and (*c*) expanded spectrum around the carrier frequency.

Modulation factor $\mu=2.0$

3.1 Amplitude Modulation

❑ Envelope detection (way1)

- AM wave can be demodulated by the *envelope detector* if
  - The AM wave is narrowband, which means that the carrier frequency is large compared to the message bandwidth
  - The percentage modulation in the AM wave is less than 100 percent.
- Envelope detector consisting of a diode and RC filter
  - The capacitor C charges rapidly and discharges slowly through R_l.

$$(r_f + R_s)C \ll \frac{1}{f_c}$$

charging time constant

$$\frac{1}{f_c} \ll R_l C \ll \frac{1}{W}$$

discharging time constant

[AM radio broadcasting]

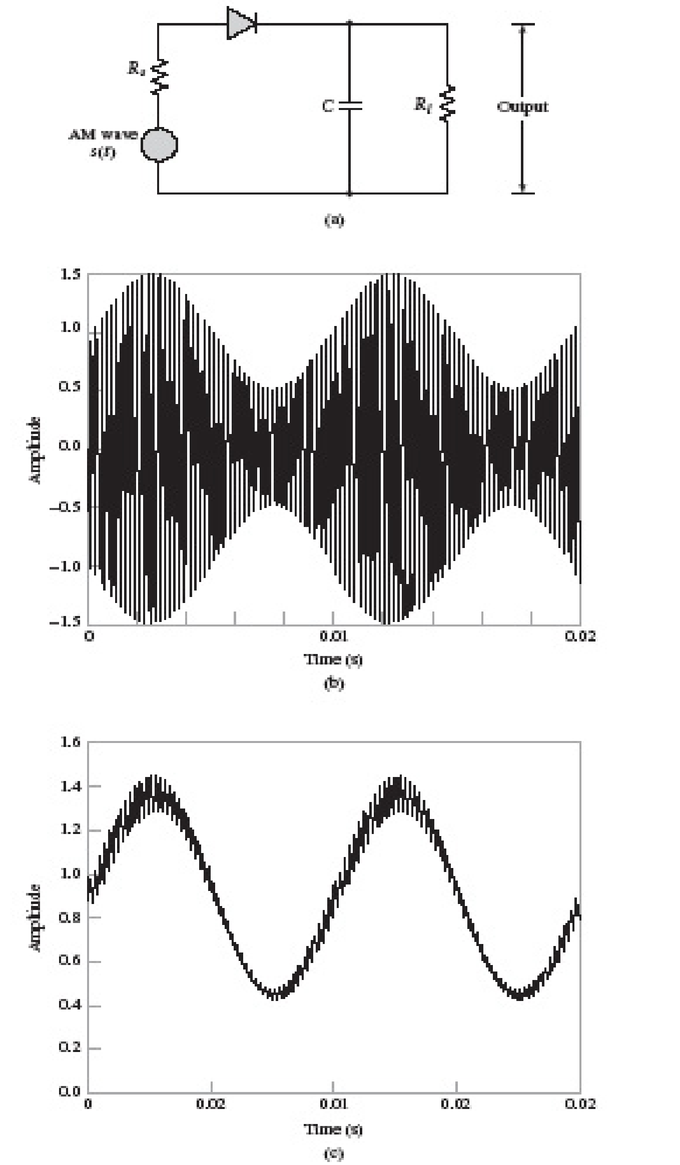
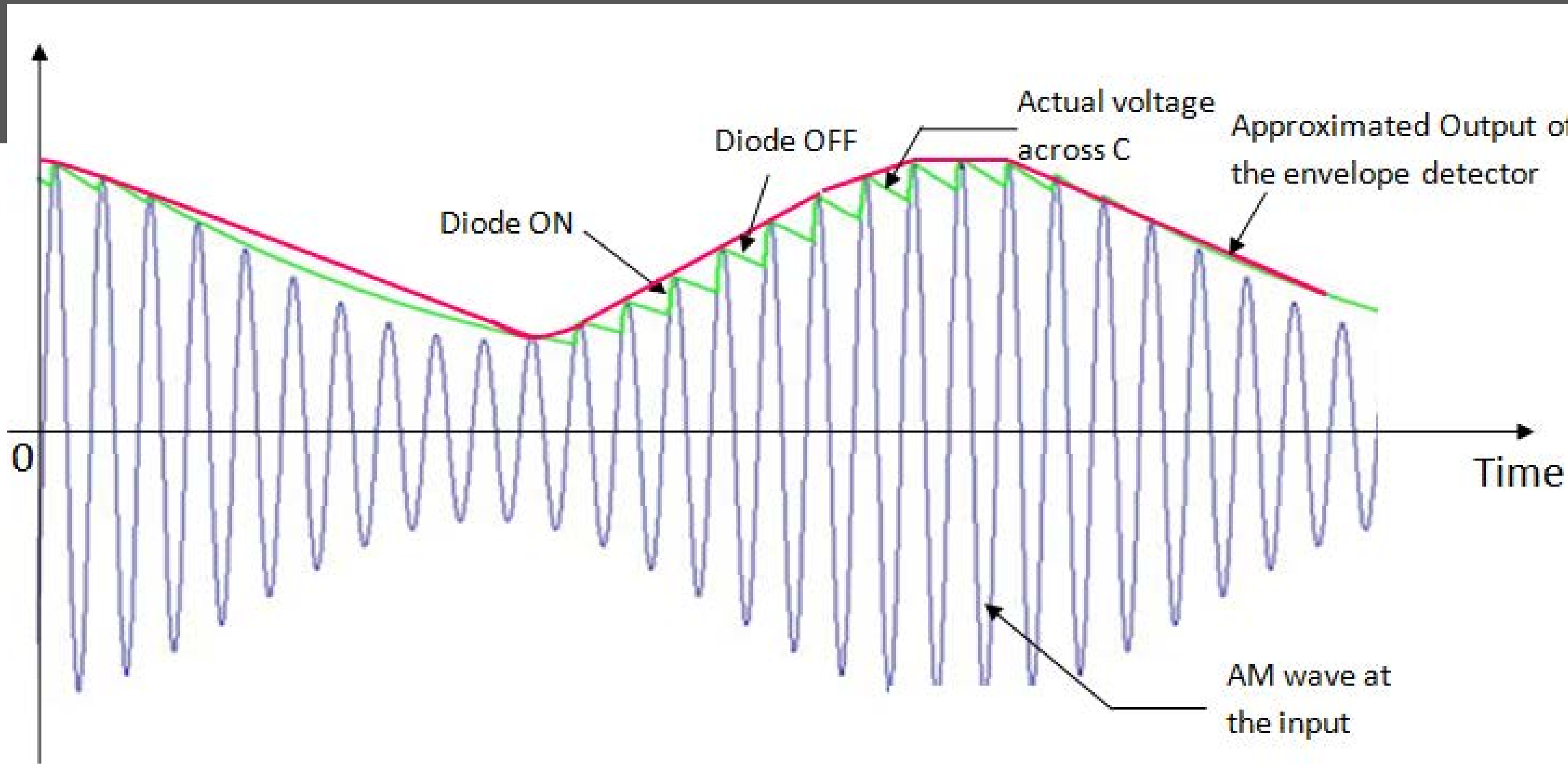
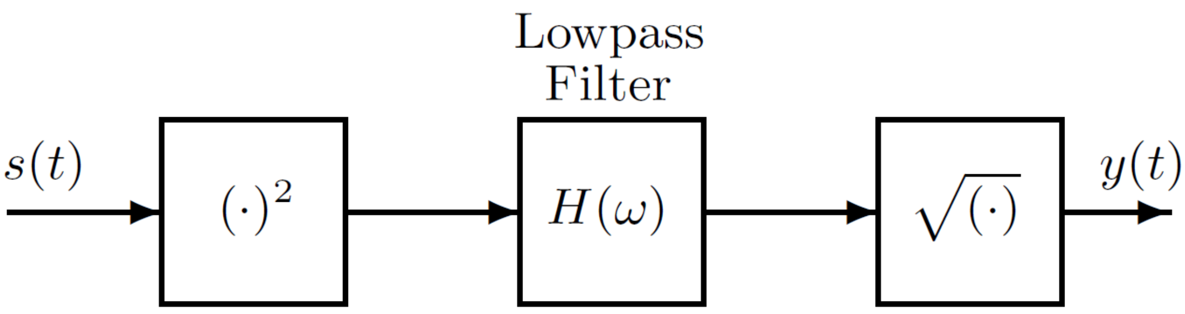


FIGURE 3.9 Envelope detector. (*a*) Circuit diagram. (*b*) AM wave input. (*c*) Envelope detector output



The capacitor charges through D and R_s when the diode i on and it discharges through R when the diode is off.
The charging time constant R_sC should be short compared to the carrier period $1/f_c$.
Thus, $R_sC << 1/f_c$
On the other hand, the discharging time constant RC should be long enough so that the capacitor discharges slowly through the load resistance R . But, this time constant should not be too long which will not allow the capacitor voltage to discharge at the maximum rate of change of the envelope .
Therefore, $1/f_c << RC << 1/W$
where, W = Maximum modulating frequency

❑ Demodulation: Square-law demodulation (way2)



Square-Law Envelope Detector

The squarer output is

$$
\begin{aligned}
s^2(t) &= A_c^2[1 + k_a m(t)]^2 \cos^2 \omega_c t \\
&= 0.5A_c^2[1 + k_a m(t)]^2 + 0.5A_c^2[1 + k_a m(t)]^2 \cos 2\omega_c t
\end{aligned}
$$

3.2 Virtues, Limitations, and Modifications of Amplitude Modulation

Practical Limitation

- AM is wasteful of transmitted power
  - The transmission of the carrier wave $c(t)$ is independent of $m(t)$.
- AM is wasteful of channel bandwidth
  - Only one sideband is necessary for the transmission of information
    - The communication channel needs to provide only the same bandwidth as the message signal.
  - AM requires a transmission bandwidth equal to twice the message bandwidth.

Three modifications of AM

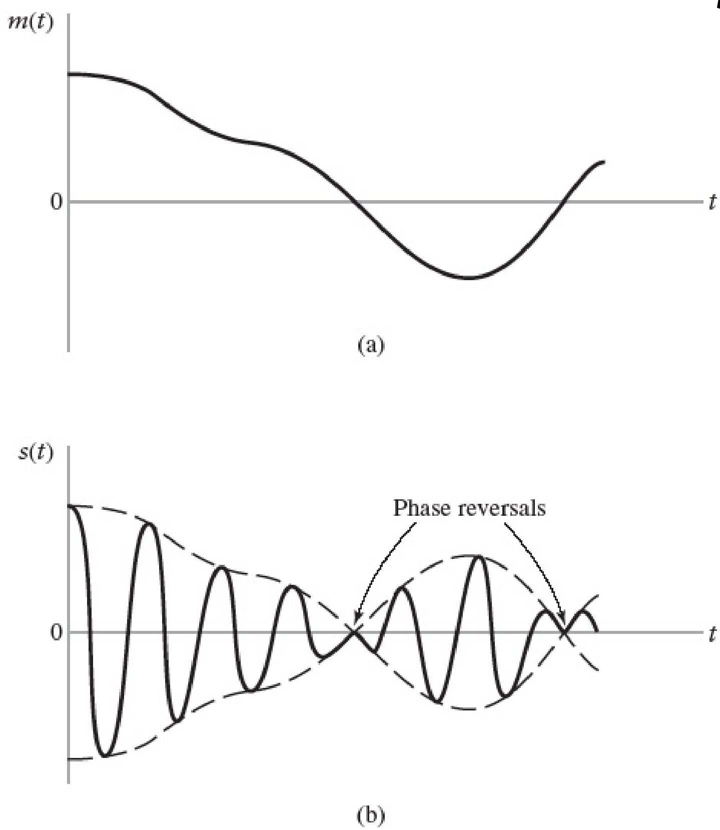
- Double sideband-suppressed carrier (DSB-SC) modulation
- Single sideband (SSB) modulation
- Vestigial sideband (VSB) modulation

3.3 Double Sideband-Suppressed Carrier Modulation

❑ Theory

- DSB-SC modulation consists of the product of the message signal and the carrier wave: (product modulator)

$$s(t) = c(t)m(t)$$
$$= A_c m(t) \cos(2\pi f_c t) \quad (3.8)$$



- Modulated signal *s*(*t*) undergoes a phase reversal whenever *m*(*t*) crosses zero.
- Envelope of DSB-SC is different from the message signal
- Simple demodulation using an envelope detection is not a viable option.

FIGURE 3.10 (*a*) Message signal $m(t)$. (*b*) DSB-SC modulated wave $s(t)$.

3.3 Double Sideband-Suppressed Carrier Modulation

- Fourier transform of *s*(*t*)

$$S(f)=\frac{1}{2}A_c[M(f-f_c)+M(f+f_c)] \quad (3.9)$$

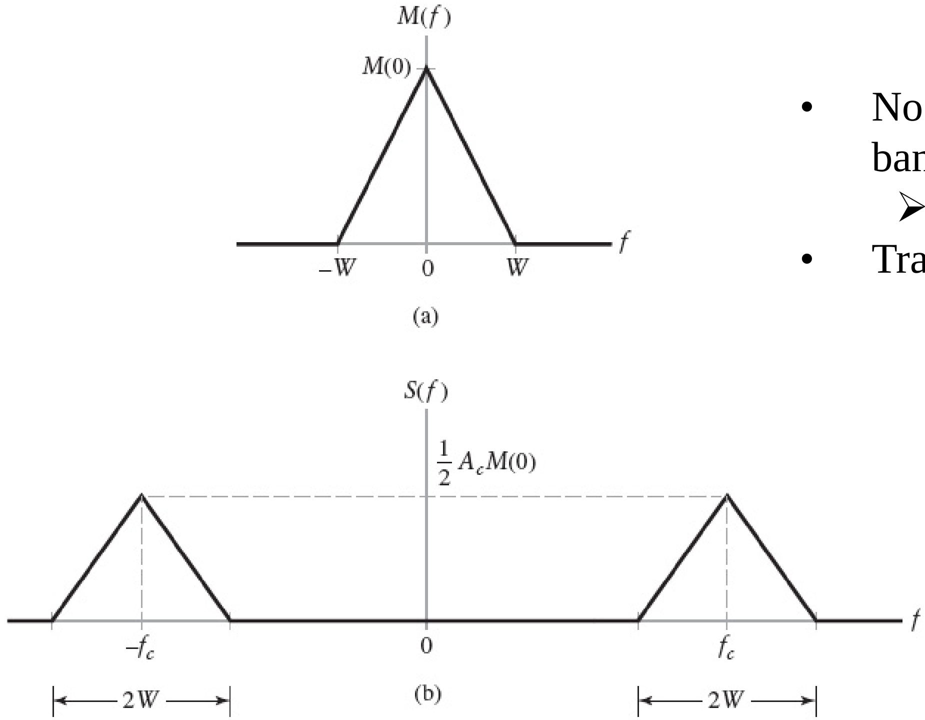


FIGURE 3.11 (*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of DSB-SC modulated wave $s(t)$.

- No advantage over AM in view of bandwidth occupancy
  - $B_T = 2W$
- Transmission power is saved over AM.

3.3 Double Sideband-Suppressed Carrier Modulation

EXAMPLE 3.2 Sinusoidal DSB-SC spectrum

Consider DSB-SC modulation using a sinusoidal modulating wave of amplitude A_m and frequency f_m and operating on a carrier of amplitude A_c and frequency f_c. The message spectrum is

$$M(f) = \frac{1}{2}A_m\delta(f - f_m) + \frac{1}{2}A_m\delta(f + f_m)$$

Invoking Eq. (3.9), the shifted spectrum $\frac{1}{2}A_cM(f - f_c)$ defines the two side-frequencies for positive frequencies:

$$\frac{1}{4}A_cA_m\delta(f - (f_c + f_m)); \qquad \frac{1}{4}A_cA_m\delta(f - (f_c - f_m))$$

The other shifted spectrum of Eq. (3.9)—namely, $\frac{1}{2}A_cM(f + f_c)$,—defines the remaining two side-frequencies for negative frequencies:

$$\frac{1}{4}A_cA_m\delta(f + (f_c - f_m)); \qquad \frac{1}{4}A_cA_m\delta(f + (f_c + f_m))$$

which are the *images* of the first two side-frequencies with respect to the origin, in reverse order.

❑ Coherent detection (synchronous demodulation)

- Recovery of the message signal $m(t)$
  1. Multiply the modulated signal $s(t)$ with a locally generated sinusoidal wave $A_c' \cos(2\pi f_c t + \phi)$
  2. Low-pass filter the product (filter output), i.e., $A_c' \cos(2\pi f_c t + \phi) \cdot s(t)$

$$s(t) = c(t)m(t)$$
$$= A_c m(t) \cos(2\pi f_c t) \quad (3.8)$$

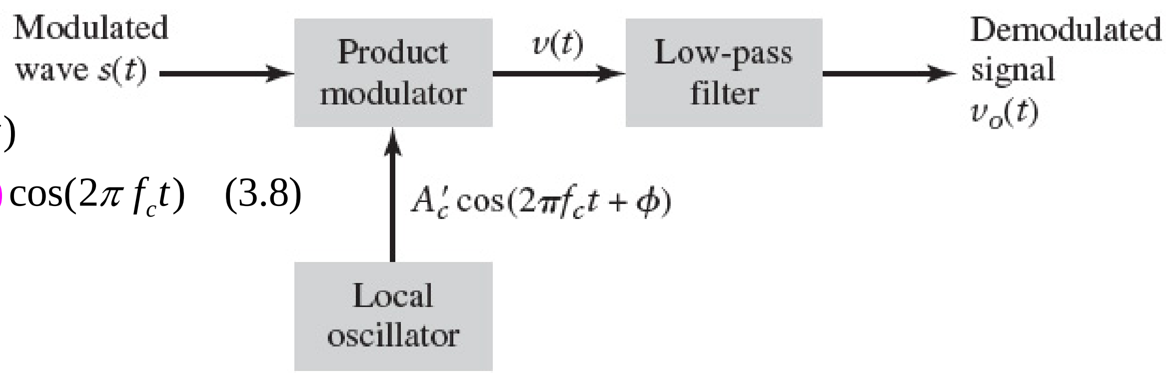


FIGURE 3.12 Block diagram of coherent detector, assuming that the local oscillator is out of phase by ϕ with respect to the sinusoidal carrier oscillator in the transmitter.

3.3 Double Sideband-Suppressed Carrier Modulation

- Product modulation output

$$v(t) = A_c' \cos(2\pi f_c t + \phi) s(t)$$
$$= A_c A_c' \cos(2\pi f_c t)\cos(2\pi f_c t + \phi) m(t)$$
$$= \frac{1}{2} A_c A_c' \cos(4\pi f_c t + \phi) m(t) + \frac{1}{2} A_c A_c' \cos(\phi) m(t) \quad (3.10)$$

where we used $\cos(\theta_1)\cos(\theta_2) = \frac{1}{2}\cos(\theta_1 + \theta_2) + \frac{1}{2}\cos(\theta_1 - \theta_2)$

- Low-pass filter output (by choosing $f_c > W$)

$$v_o(t) = \frac{1}{2} A_c A_c' \cos(\phi) m(t) \quad (3.11)$$

  - $v_o(t)$ is proportional to $m(t)$
  - The quadrature null effect : $v_o(t) = 0$ for $\phi = \pm\pi/2$
    - The phase error ϕ in the local oscillator (frequency offset) causes the detector output to be attenuated by a factor equal to $\cos\phi$
    - The local oscillator in the receiver and the carrier wave must be synchronized in frequency and phase. → increasing complexity

3.3 Double Sideband-Suppressed Carrier Modulation

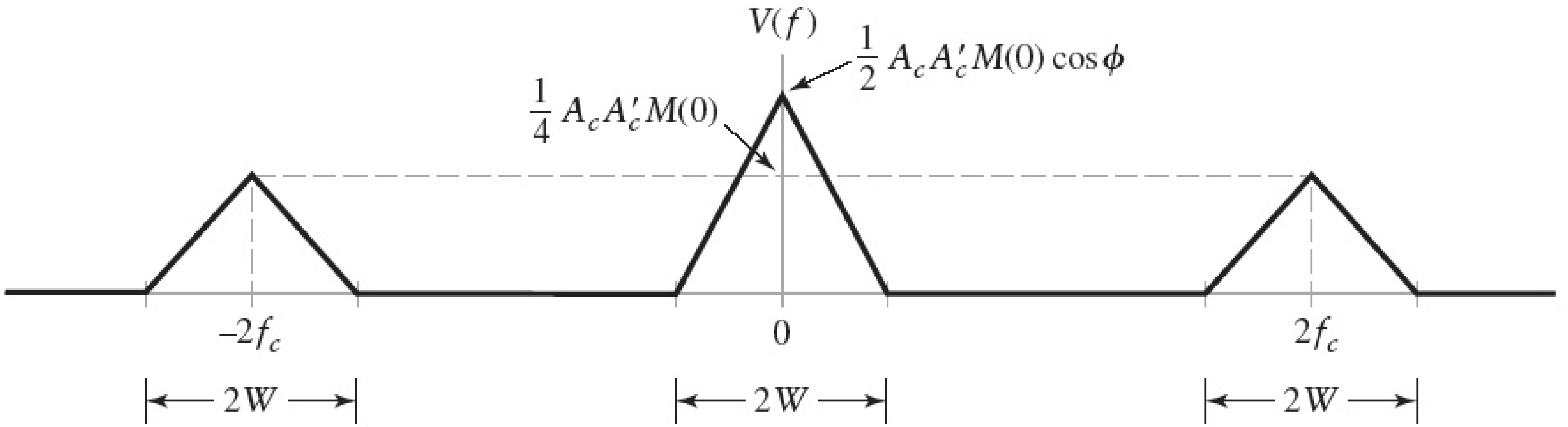


FIGURE 3.13 Illustration of the spectrum of product modulator output $v(t)$ in the coherent detector of Fig. 3.12, which is produced in response to a DSB-SC modulated wave as the detector input.

3.4 Costas Receiver

Costas Receiver

- Consists of two coherent detectors supplied with the same input signal
  - Two local oscillator signals are in phase quadrature with respect to each other
  - The frequency of local oscillator = f_c (*a priori*)
  - I-channel : in-phase (cos) coherent detector
  - Q-channel : quadrature-phase (sin) coherent detector
- Two detectors form a negative feedback to maintain the local oscillator in synchronism with the carrier wave.

3.4 Costas Receiver

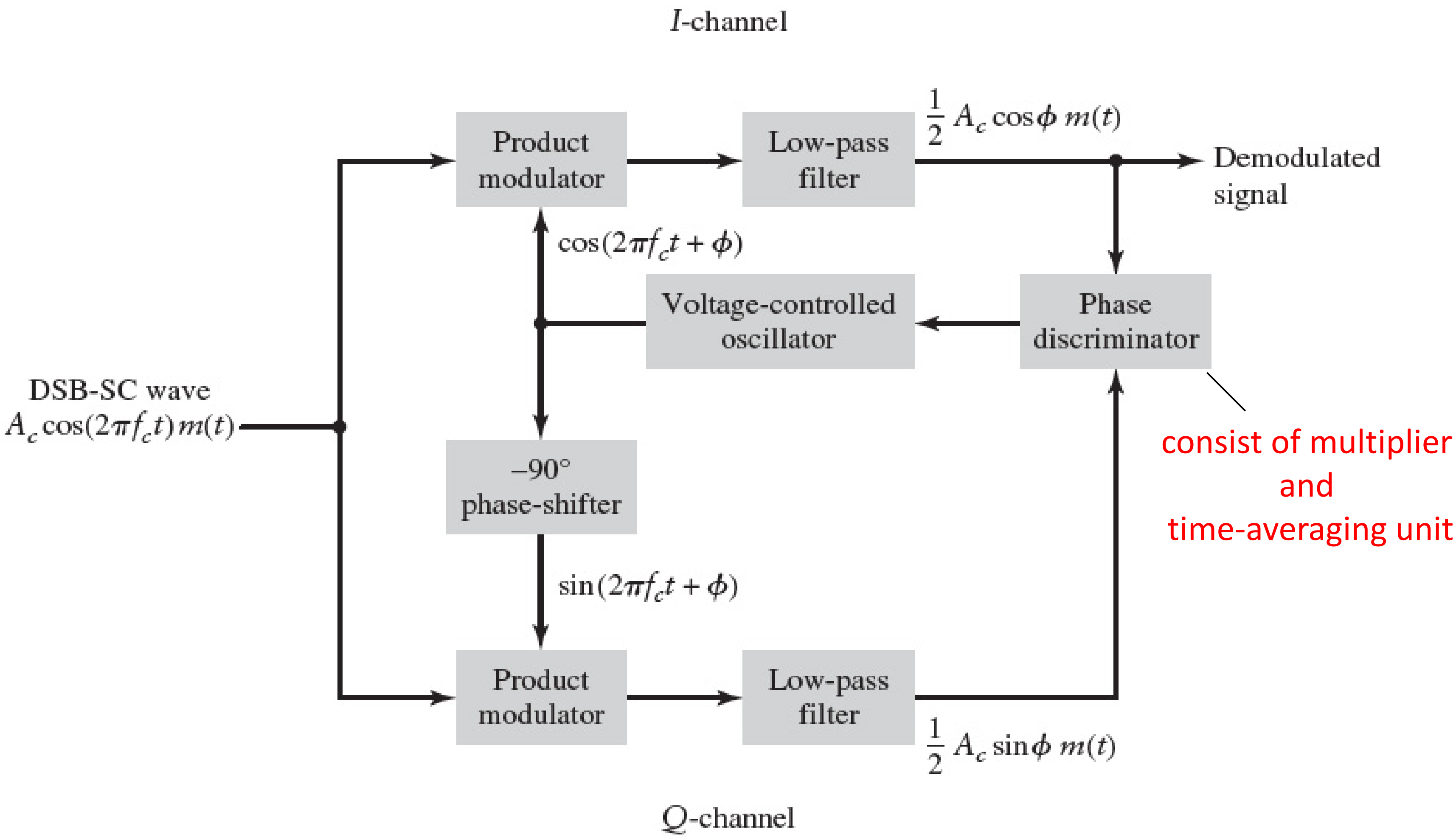


FIGURE 3.16 Costas receiver for the demodulation of a DSB-SC modulated wave.

3.5 Quadrature-Carrier Multiplexing

- Quadrature-Amplitude modulation (**QAM**)
  - This scheme enables two DSB-SC modulated waves to occupy the same channel bandwidth (exploiting "quadrature null effect") as one DSB-SC wave

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t) \quad (3.12)$$

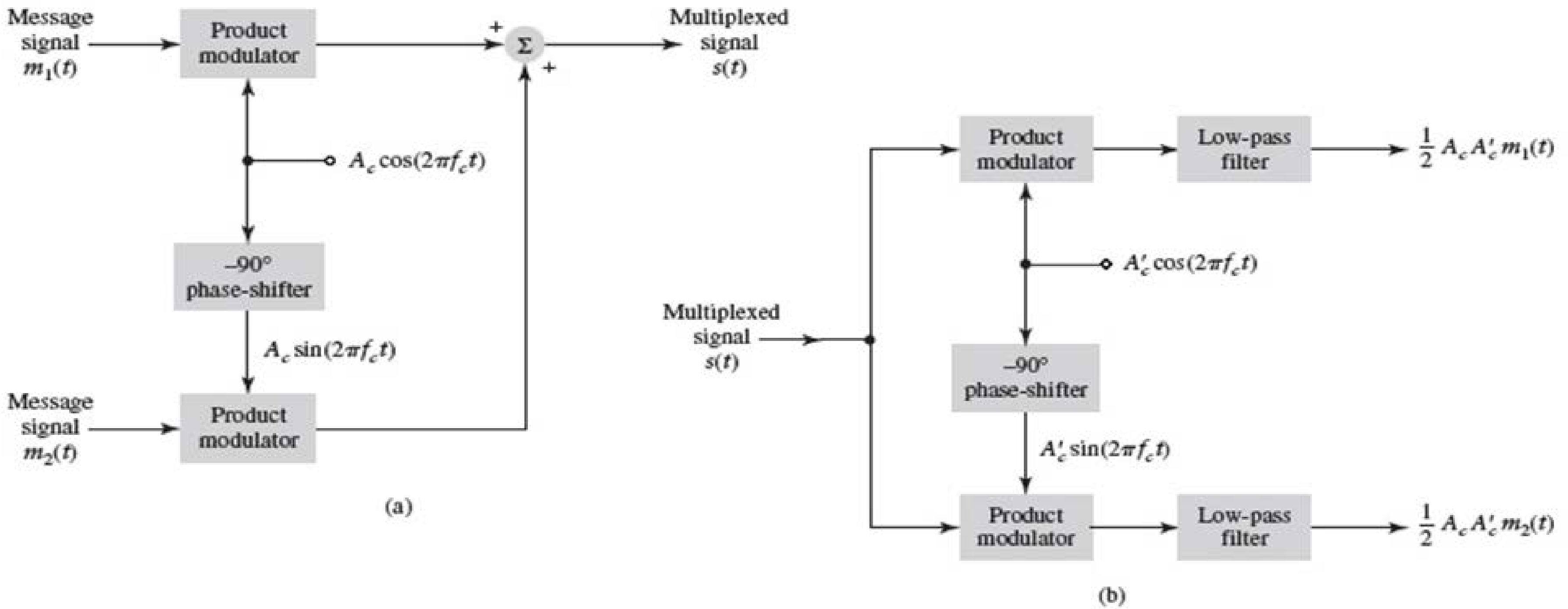


FIGURE 3.17 Quadrature-carrier multiplexing system: (*a*) Transmitter, (*b*) receiver.

3.6 Single-Sideband Modulation

- Single-Sideband Modulation
  - Suppress one of the two sideband in the DSB-SC modulated wave
- Theory
  - A DSB-SC modulator using the sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

  - The resulting DSB-SC modulated wave is

$$\begin{aligned} S_{DSB}(t) &= c(t)m(t) \\ &= A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) \\ &= \frac{1}{2} A_c A_m \cos[2\pi(f_c + f_m)t] + \frac{1}{2} A_c A_m \cos[2\pi(f_c - f_m)t] \quad (3.13) \end{aligned}$$

Upper SSB ($f_c + f_m$), Lower SSB ($f_c - f_m$): two side-frequencies

  - Suppressing the second term in Eq. (3.13), the upper and lower SSB modulated wave are

$$\begin{aligned} S_{USSB}(t) &= \frac{1}{2} A_c A_m \cos[2\pi(f_c + f_m)t] \quad (3.14) \\ &= \frac{1}{2} A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) - \frac{1}{2} A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t) \quad (3.15) \end{aligned}$$

3.6 Single-Sideband Modulation

- Suppose we suppress the first term (high side-frequency) in (3.13)
  - Lower SSB modulated wave:

$$S_{LSSB}(t) = \frac{1}{2}A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) + \frac{1}{2}A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t) \quad (3.16)$$

- A sinusoidal SSB modulated wave

$$S_{SSB}(t) = \frac{1}{2}A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) \mp \frac{1}{2}A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t) \quad (3.17)$$

- For a periodic message signal *m*(*t*) defined by the Fourier series,

$$m(t) = \sum_n a_n \cos(2\pi f_n t) \quad (3.18)$$

the SSB modulated wave is

Wideband phase shifter:= Hilbert transform

$$S_{SSB}(t) = \frac{1}{2}A_c \cos(2\pi f_c t)\sum_n a_n \cos(2\pi f_n t) \mp \frac{1}{2}A_c \sin(2\pi f_c t)\sum_n a_n \sin(2\pi f_n t) \quad (3.19)$$

- For another periodic signal, $\hat{m}(t) = \sum_n a_n \sin(2\pi f_n t) \quad (3.20)$

the SSB modulated wave is $S_{SSB}(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t) \quad (3.21)$

3.6 Single-Sideband Modulation

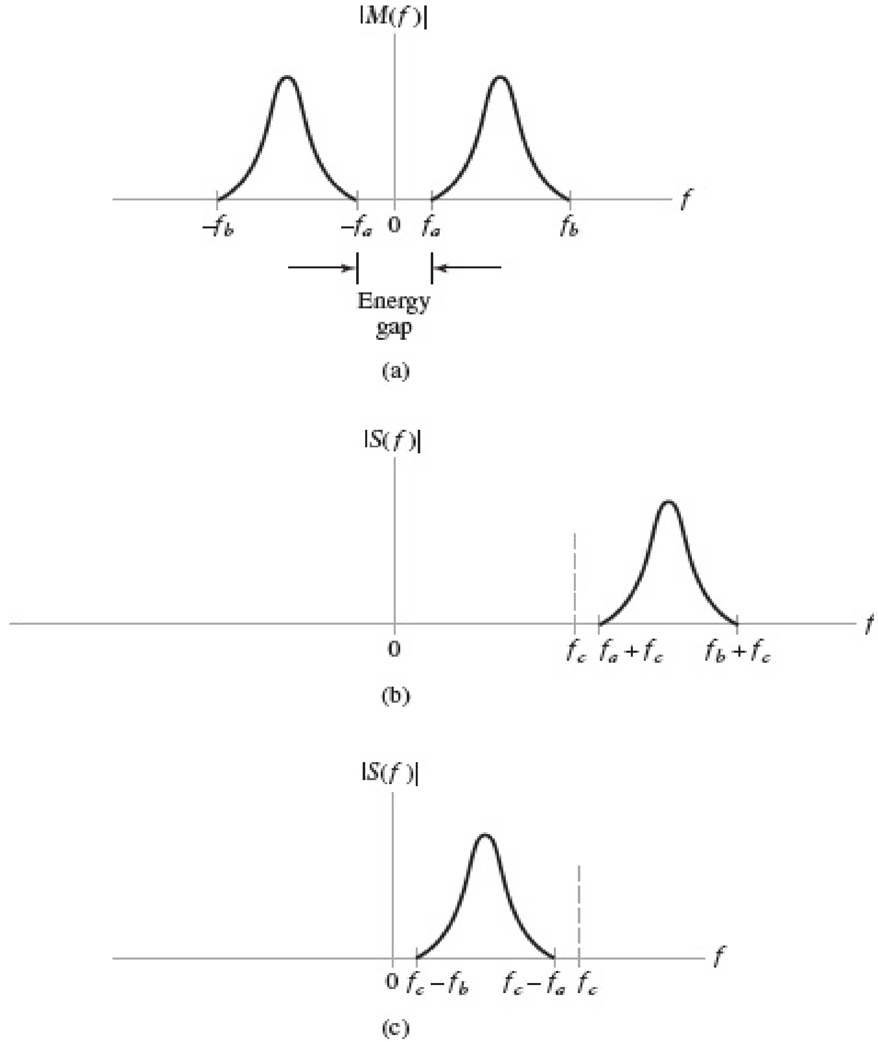


FIGURE 3.18 (*a*) Spectrum of a message signal $m(t)$ with energy gap centered around zero frequency. Corresponding spectra of SSB-modulated waves using (*b*) upper sideband, and (*c*) lower sideband. In parts (*b*) and (*c*), the spectra are only shown for positive frequencies.

3.6 Single-Sideband Modulation

❑ Modulators for SSB

- Frequency Discrimination Method

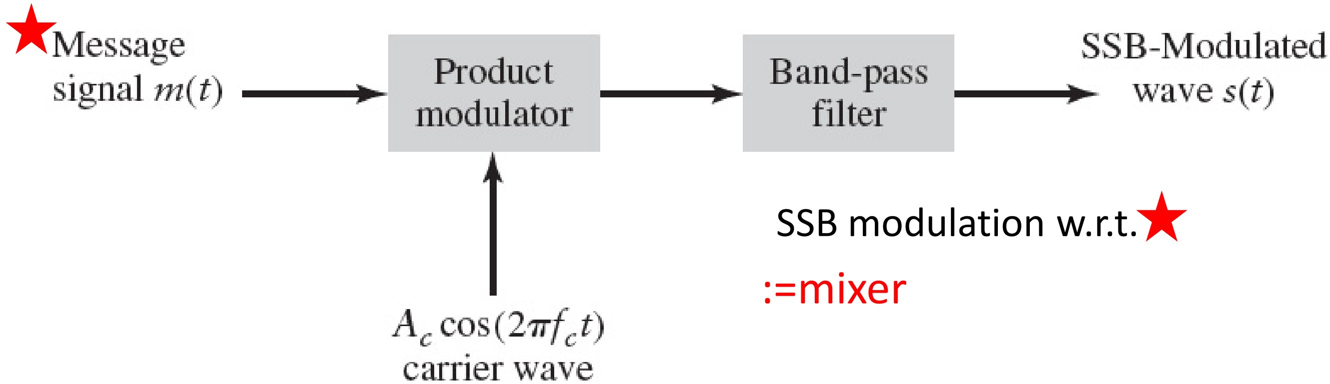


FIGURE 3.19 Frequency-discrimination scheme for the generation of a SSB modulated wave.

- For the design of the band-pass filter to be practically feasible, there must be a certain separation between the two sidebands that is wide enough to accommodate the transition band of the band-pass filter.

Phase Discrimination Method

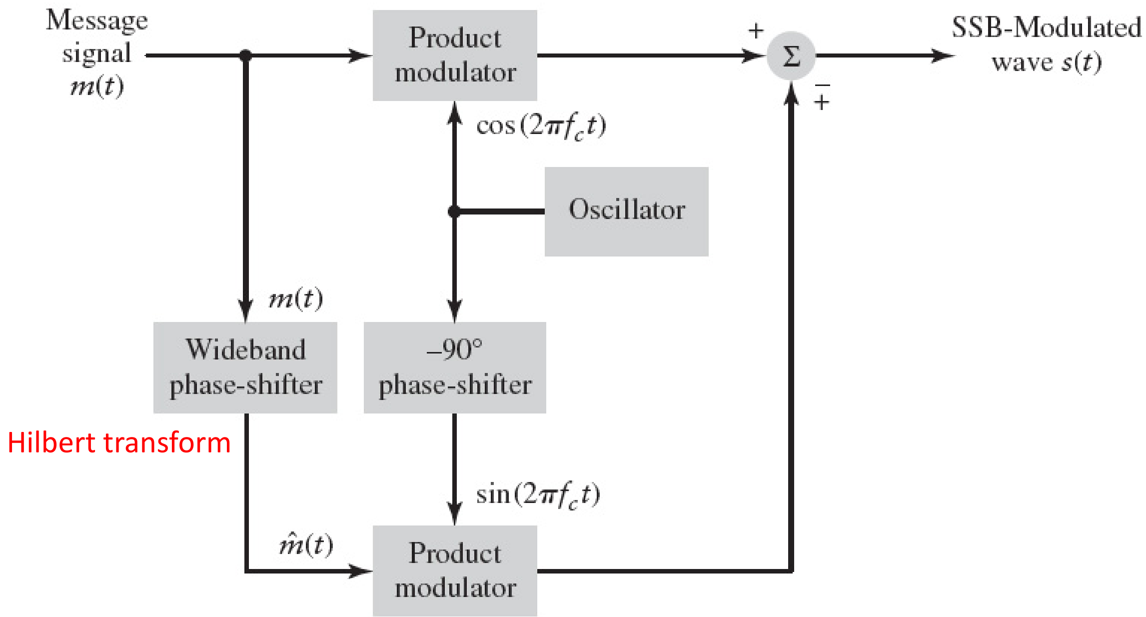


FIGURE 3.20 Phase discrimination method for generating a SSB-modulated wave. Note: The plus sign at the summing junction pertains to transmission of the lower sideband and the minus sign pertains to transmission of the upper sideband.

Hilbert transform

$$H(u)(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{u(\tau)}{t-\tau}\, d\tau$$

Signal $u(t)$	Hilbert transform[fn 1] $H(u)(t)$
$\sin(\omega t)$ [fn 2]	$\operatorname{sgn}(\omega)\sin\left(\omega t - \frac{\pi}{2}\right) = -\operatorname{sgn}(\omega)\cos(\omega t)$
$\cos(\omega t)$ [fn 2]	$\operatorname{sgn}(\omega)\cos\left(\omega t - \frac{\pi}{2}\right) = \operatorname{sgn}(\omega)\sin(\omega t)$
$e^{i\omega t}$	$\operatorname{sgn}(\omega)e^{i\left(\omega t - \frac{\pi}{2}\right)} = -i\cdot\operatorname{sgn}(\omega)e^{i\omega t}$
$\frac{1}{t^2+1}$	$\frac{t}{t^2+1}$
e^{-t^2}	$2\pi^{-1/2}F(t)$ (see Dawson function)
Sinc function $\frac{\sin(t)}{t}$	$\frac{1-\cos(t)}{t}$
Rectangular function $\sqcap(t)$	$\frac{1}{\pi}\ln\left\lvert\frac{t+\frac{1}{2}}{t-\frac{1}{2}}\right\rvert$
Dirac delta function $\delta(t)$	$\frac{1}{\pi t}$
Characteristic Function $\chi_{[a,b]}(t)$	$\frac{1}{\pi}\ln\left\lvert\frac{t-a}{t-b}\right\rvert$

- Wideband phase shifter: interfere with the in-phase path so as to eliminate power in one of the two sidebands, depending on whether upper SSB or lower SSB is the requirement.

3.6 Single-Sideband Modulation

❑ Coherent Detection of SSB

- Synchronization (frequency and phase) of a local oscillator in the receiver with the oscillator generating the carrier in the transmitter
- The demodulation of SSB is further complicated by the additional suppression of the upper or lower sideband.

❑ Frequency Translation

- Single sideband modulation is in fact a form of frequency translation
  - Frequency changing
  - Mixing
  - Heterodyning

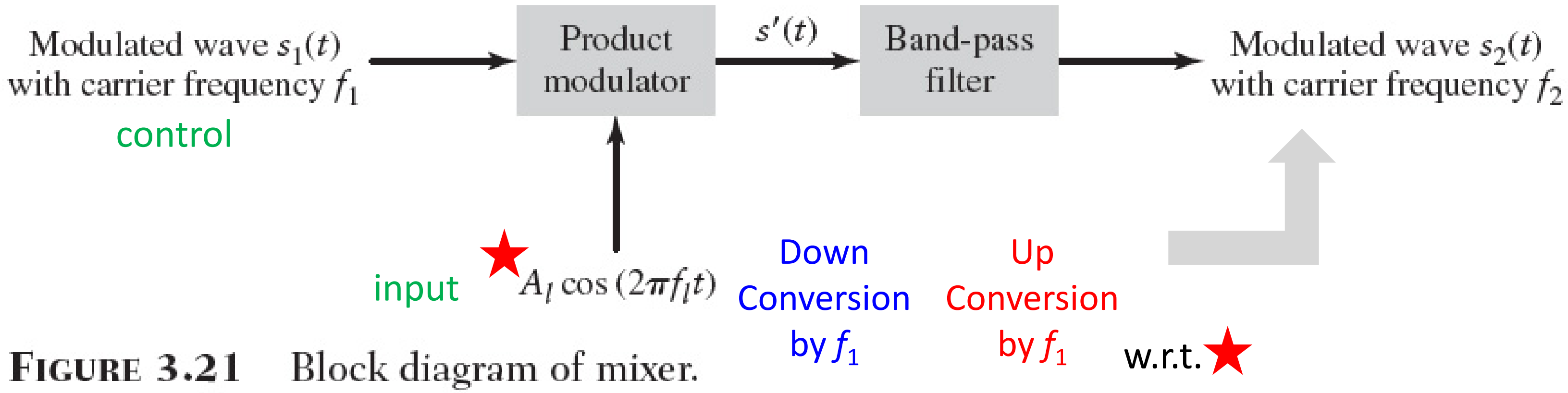


FIGURE 3.21 Block diagram of mixer.

3.6 Single-Sideband Modulation

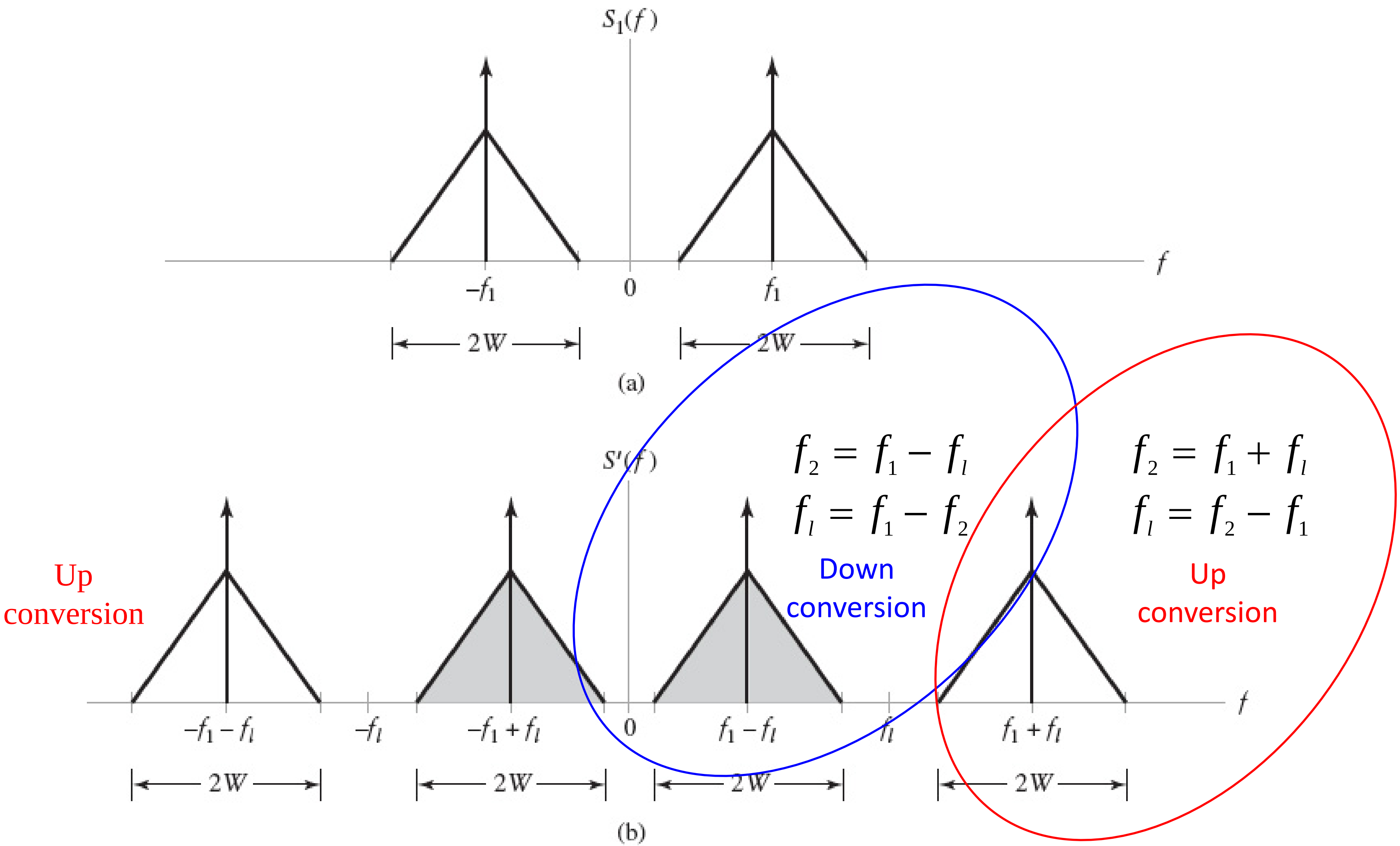


FIGURE 3.22 (*a*) Spectrum of modulated signal $s_1(t)$ at the mixer input. (*b*) Spectrum of the corresponding signal $s'(t)$ at the output of the product modulator in the mixer.

3.7 Vestigial Sideband Modulation

- For the spectrally efficient transmission of wideband signals
  - Typically, the spectra of wideband signals contain significantly low frequencies → the use of SSB modulation is impractical.
  - The spectral characteristics of wideband data befit the use of DSB-SC. However, DSB-SC requires a transmission bandwidth equal to twice the message bandwidth → violate the bandwidth conservation requirement.
- Vestigial sideband (VSB) modulation
  - Instead of completely removing a sideband, a trace of vestige of that sideband is transmitted → the name *"vestigial sideband"*
  - Instead of transmitting the other sideband in full, almost the whole of this second band is also transmitted.
  - Transmission bandwidth

$$B_T = f_v + W$$

where f_v : vestige bandwidth

W : message bandwidth

3.7 Vestigial Sideband Modulation

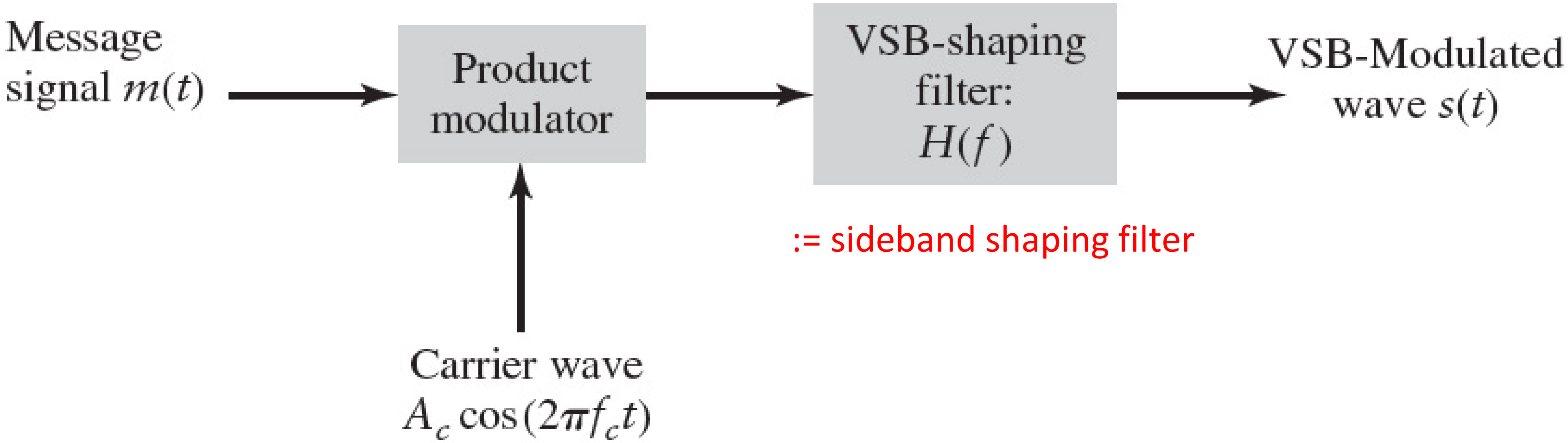


FIGURE 3.23 VSB modulator using frequency discrimination.

3.7 Vestigial Sideband Modulation

❑ Sideband Shaping Filter

- The transmitted vestige compensates for the spectral portion missing from the other sideband.
- Two properties of the sideband shaping filter
  1. Coherent detection recovers a replica of the message signal.

$$H(f+f_c)+H(f-f_c)=1, \quad \text{for } -W \le f \le W \qquad (3.26)$$

  2. The transfer function of the sideband shaping filter exhibits odd symmetry about the carrier frequency

Let $H(f)=u(f-f_c)-H_v(f-f_c), \quad \text{for } f_c - f_v < |f| < f_c + W \qquad (3.27)$

$$H_v(-f)=-H_v(f) \qquad (3.29)$$ odd symmetry

3.7 Vestigial Sideband Modulation

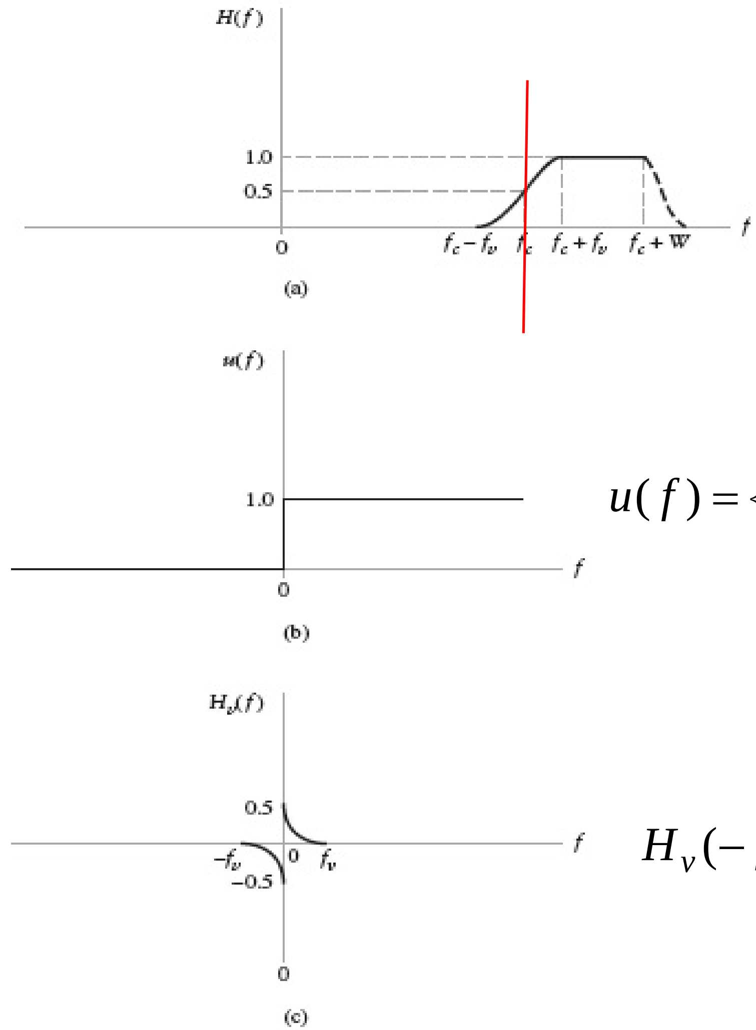


FIGURE 3.24 (*a*) Amplitude response of sideband-shaping filter; only the positive-frequency portion is shown, the dashed part of the amplitude response is arbitrary. (*b*) Unit-step function defined in the frequency domain. (*c*) Low-pass transfer function $H_v(f)$.

$$u(f) = \begin{cases} 1, & \text{for } f > 0 \\ 0, & \text{for } f < 0 \end{cases} \quad (3.28)$$

$$H_v(-f) = -H_v(f) \quad (3.29)$$

3.7 Vestigial Sideband Modulation

EXAMPLE 3.3 Sinusoidal VSB

Consider the simple example of sinusoidal VSB modulation produced by the sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

and carrier wave

$$c(t) = A_c \cos(2\pi f_c t)$$

Let the upper side-frequency at $f_c + f_m$ as well as its image at $-(f_c + f_m)$ be attenuated by the factor k. To satisfy the condition of Eq. (3.26), the lower side-frequency at $f_c - f_m$ and its image $-(f_c - f_m)$ must be attenuated by the factor $(1 - k)$. The VSB spectrum is therefore

$$S(f) = \frac{1}{4}kA_cA_m[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))]$$
$$+\frac{1}{4}(1 - k)A_cA_m[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))]$$

Plot it

3.7 Vestigial Sideband Modulation

Correspondingly, the sinusoidal VSB modulated wave is defined by

$$s(t) = \frac{1}{4}kA_cA_m[\exp(j2\pi(f_c + f_m)t) + \exp(-j2\pi(f_c + f_m)t)]$$

$$+\frac{1}{4}(1 - k)A_cA_m[\exp(j2\pi(f_c - f_m)t) + \exp(-j2\pi(f_c - f_m)t)]$$

$$= \frac{1}{2}kA_cA_m \cos(2\pi(f_c + f_m)t) + \frac{1}{2}(1 - k)A_cA_m \cos(2\pi(f_c - f_m)t) \quad (3.30)$$

Using well-known trigonometric identities to expand the cosine terms $\cos(2\pi(f_c + f_m)t)$ and $\cos(2\pi(f_c - f_m)t)$, we may reformulate Eq. (3.30) as the linear combination of two sinusoidal DSB-SC modulated waves.

k=0.5 → DSB-SC
k=0 → L-SSB
k=1 → U-SSB
o.w. → VSB

$$s(t) = \frac{1}{2}A_cA_m \cos(2\pi f_c t) \cos(2\pi f_m t)$$

$$+\frac{1}{2}A_cA_m(1 - 2k) \sin((2\pi f_c t) \sin(2\pi f_m t)) \quad (3.31)$$

where the first term on the right-hand side is the in-phase component of $s(t)$ and the second term is the quadrature component.

3.7 Vestigial Sideband Modulation

❑ Coherent Detection of VSB

- The demodulation consists of
  1. multiplying $s(t)$ with a locally generated sinusoid $c(t) = A_c'\cos(2\pi f_c t)$
  2. low-pass filtering the product signal $v(t)=s(t)\cdot c(t)$

- Fourier transform of the product signal $v(t) = A_c' s(t)\cos(2\pi f_c t)$ is

$$V(f) = \frac{1}{2}A_c'[S(f - f_c) + S(f + f_c)] \quad (3.32)$$

  – Fourier transform of VSB modulated wave $s(t)$

$$S(f) = \frac{1}{2}A_c[M(f - f_c) + M(f + f_c)]H(f) \quad (3.33)$$

  – Shifting the VSB spectrum to the right and left by f_c

$$S(f - f_c) = \frac{1}{2}A_c[M(f - 2f_c) + M(f)]H(f - f_c) \quad (3.34)$$

$$S(f + f_c) = \frac{1}{2}A_c[M(f) + M(f + 2f_c)]H(f + f_c) \quad (3.35)$$

3.7 Vestigial Sideband Modulation

- Hence,

$$V(f) = \frac{1}{4} A_c A_c' M(f)[H(f - f_c) + H(f + f_c)]$$

$$+ \frac{1}{4} A_c A_c' [M(f - 2f_c)H(f - f_c) + M(f + 2f_c)H(f + f_c)]$$

By $H(f + f_c) + H(f - f_c) = 1, \quad \text{for} -W \le f \le W \quad (3.26)$

$$V(f) = \underbrace{\frac{1}{4} A_c A_c' M(f)}_{\text{Scaled version of } m(t)} + \underbrace{\frac{1}{4} A_c A_c' [M(f - 2f_c)H(f - f_c) + M(f + 2f_c)H(f + f_c)]}_{\text{High frequency component}} \quad (3.36)$$

Scaled version of *m*(*t*)

High frequency component

- The low-pass filter in the coherent detector has a cutoff frequency just slightly greater than the message bandwidth
  - High frequency component of *v*(*t*) is removed.
  - The resulting demodulated signal is a scaled version of the desired message signal *m*(*t*).

3.7 Vestigial Sideband Modulation

EXAMPLE 3.4 Coherent detection of sinusoidal VSB

Recall from Eq. (3.31) of Example 3.3, that the sinusoidal VSB modulated signal is defined by

$$s(t) = \frac{1}{2}A_cA_m\cos(2\pi f_mt)\cos(2\pi f_ct) + \frac{1}{2}A_cA_m(1-2k)\sin(2\pi f_ct)\sin(2\pi f_mt)$$

Multiplying $s(t)$ by $A_c'\cos(2\pi f_ct)$ in accordance with perfect coherent detection yields the product signal

$$\begin{aligned} v(t) &= A_c's(t)\cos(2\pi f_ct) \\ &= \frac{1}{2}A_cA_c'A_m\cos(2\pi f_mt)\underline{\cos^2(2\pi f_ct)} \\ &\quad + \frac{1}{2}A_cA_c'A_m(1-2k)\sin(2\pi f_mt)\underline{\sin(2\pi f_ct)\cos(2\pi f_ct)} \end{aligned}$$

Next, using the trigonometric identities

$$\cos^2(2\pi f_ct) = \frac{1}{2}[1+\cos(4\pi f_ct)]$$

3.7 Vestigial Sideband Modulation

and

$$\sin(2\pi f_c t)\cos(2\pi f_c t) = \frac{1}{2}\sin(4\pi f_c t)$$

we may redefine $\nu(t)$ as

$$\nu(t) = \frac{1}{4}A_c A_c' A_m \cos(2\pi f_m t) + \frac{1}{4}A_c A_c' A_m[\cos(2\pi f_m t)\cos(4\pi f_c t) + (1 - 2k)\sin(2\pi f_m t)\sin(4\pi f_c t)] \quad (3.37)$$

The first term on the right-hand side of Eq. (3.37) is a scaled version of the message signal $A_m \cos(2\pi f_m t)$. The second term of the equation is a new sinusoidal VSB wave modulated onto a carrier of frequency $2f_c$, which represents the high-frequency components of $\nu(t)$. This second term is removed by the low-pass filter in the detector of Fig. 3.12, provided that the cut-off frequency of the filter is just slightly greater than the message frequency f_m.

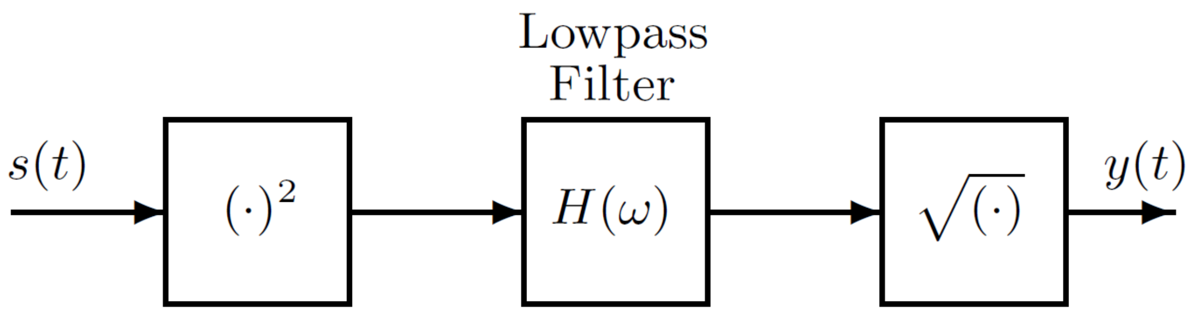
3.7 Vestigial Sideband Modulation

Example 3.5 Envelope detection of VSB plus carrier

The coherent detection of VSB requires synchronism of the receiver to the transmitter, which increases system complexity. To simplify the demodulation process, we may purposely add the carrier to the VSB signal (scaled by the factor k_a) prior to transmission and then use envelope detection in the receiver.[3] Assuming sinusoidal modulation, the "VSB-plus-carrier" signal is defined [see Eq. (3.31) of Example 3.3) as

$$
\begin{aligned}
s_{\mathrm{VSB+C}}(t) &= A_c \cos(2\pi f_c t) + k_a s(t), \qquad k_a = \text{amplitude sensitivity factor} \\
&= A_c \cos(2\pi f_c t) + \frac{k_a}{2} A_c A_m \cos(2\pi f_m t)\cos(2\pi f_c t) \\
&\quad + \frac{k_a}{2} A_c A_m (1 - 2k)\sin(2\pi f_m t)\sin(2\pi f_c t) \\
&= A_c\left[1 + \frac{k_a}{2} A_m \cos(2\pi f_m t)\right]\cos(2\pi f_c t) \\
&\quad + \frac{k_a}{2} A_c A_m (1 - 2k)\sin(2\pi f_m t)\sin(2\pi f_c t)
\end{aligned}
$$

❑ Demodulation: Square-law demodulation



Square-Law Envelope Detector

c.f.: coherent detection ?

The squarer output is

$$\begin{aligned} s^2(t) &= A_c^2[1 + k_a m(t)]^2 \cos^2 \omega_c t \\ &= 0.5A_c^2[1 + k_a m(t)]^2 + 0.5A_c^2[1 + k_a m(t)]^2 \cos 2\omega_c t \end{aligned}$$

3.7 Vestigial Sideband Modulation

The envelope of $s_{\text{VSB+C}}(t)$ is therefore

$$a(t) = \left\{A_c^2\left[1 + \frac{k_a}{2}A_m\cos(2\pi f_m t)\right]^2 + A_c^2\left[\frac{k_a}{2}A_m(1-2k)\sin(2\pi f_m t)\right]^2\right\}^{1/2}$$

$$= A_c\left[1 + \frac{k_a}{2}A_m\cos(2\pi f_m t)\right]\left\{1 + \left[\frac{\frac{k_a}{2}A_m(1-2k)\sin(2\pi f_m t)}{1 + \frac{k_a}{2}A_m\cos(2\pi f_m t)}\right]^2\right\}^{1/2} \quad (3.38)$$

Equation (3.38) shows that *distortion* in the envelope detection performed on the envelope $a(t)$ is contributed by the quadrature component of the sinusoidal VSB signal. This distortion can be reduced by using a combination of two methods:

- The amplitude sensitivity factor k_a is reduced, which has the effect of reducing the percentage modulation.
- The width of the vestigial sideband is reduced, which has the effect of reducing the factor $(1 - 2k)$.

Both of these methods are intuitively satisfying in light of what we see inside the square brackets in Eq. (3.38).

Reminder: AM, DSB-SC, SSB, VSB

$$(3.2)\ s(t) = A_c[1+k_a m(t)]\cos(2\pi f_c t)$$ AM

$$(3.8)\ s(t) = A_c m(t)\cos(2\pi f_c t)$$ DSB-SC ← $k=0.5$

$$(3.21)\ s(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$ U-SSB ← $k=1$; L-SSB ← $k=0$

$$(3.33)\ S(f) = \frac{1}{2}A_c\left[M(f-f_c)+M(f+f_c)\right]H(f)$$ VSB

$$H(f) = u(f-f_c) - H_v(f-f_c),\quad \text{for } f_c - f_v < |f| < f_c + W \quad (3.27)$$

$$H_v(-f) = -H_v(f) \quad (3.29)$$

$$(3.31)\ s(t) = \frac{1}{2}A_c A_m\cos(2\pi f_m t)\cos(2\pi f_c t) + \frac{1}{2}A_c A_m(1-2k)\sin(2\pi f_m t)\sin(2\pi f_c t)$$

Sinusoidal VSB

$$s(t) = A_c\cos(2\pi f_c t) + k_a\left[(3.31)\right]$$ Sinusoidal VSB+carrier

3.8 Baseband Representation of Modulated Waves and Band-Pass Filters

Baseband

- This term is used to designate the band of frequencies representing the original signal as delivered by a source of information

Baseband Representation of Modulation Waves

- A linear modulated wave

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \quad (3.39)$$

← pass-band signals

Let $c(t) = \cos(2\pi f_c t)$: carrier wave with frequency f_c

$\hat{c}(t) = \sin(2\pi f_c t)$: *quadrature-phase* version of carrier

- The modulated wave in the compact form

$$s(t) = s_I(t)c(t) - s_Q(t)\hat{c}(t) \quad (3.40)$$

: canonical representation of linear modulated waves

$s_I(t)$: *in-phase* component

$s_Q(t)$: *quadrature* component

3.8 Baseband Representation of Modulated Waves and Band-Pass Filters

- The **complex envelope** of the modulated wave

  $$\tilde{s}(t) = s_I(t) + js_Q(t) \quad (3.41)$$ ← baseband signals

- Complex carrier wave

  $$\tilde{c}(t) = c(t) + j\hat{c}(t) = \cos(2\pi f_c t) + j\sin(2\pi f_c t)$$
  $$= \exp(j2\pi f_c t) \quad (3.42)$$

- Modulated wave

  $$s(t) = \mathrm{Re}[\tilde{s}(t)\tilde{c}(t)] = \mathrm{Re}[\tilde{s}(t)\exp(j2\pi f_c t)] \quad (3.43)$$

- The practical advantage of the complex envelope
  - The highest frequency component of $s(t)$ may be as large as f_c+W, where f_c is the carrier frequency and W is the message bandwidth
  - On the other hand, the highest frequency component of $\tilde{s}(t)$ is considerably smaller, being limited by the message bandwidth W.

3.8 ... d-

Pass...

TABLE 3.1 *Different Forms of Linear Modulation as Special Cases of Eq. (3.39), assuming unit carrier amplitude*

Type of modulation	*In-phase component* $s_I(t)$	*Quadrature component* $s_Q(t)$	*Comments*
AM	$1 + k_a m(t)$	0	k_a = amplitude sensitivity $m(t)$ = message signal
DSB-SC	$m(t)$ $m_1(t)$	0 $m_2(t)$	
SSB:			
(a) Upper sideband transmitted	$\frac{1}{2}m(t)$	$\frac{1}{2}\hat{m}(t)$	$\hat{m}(t)$ = Hilbert transform of $m(t)$ (see part (i) of footnote 4)
(b) Lower sideband transmitted	$\frac{1}{2}m(t)$	$-\frac{1}{2}\hat{m}(t)$	
VSB:			
(a) Vestige of lower sideband transmitted	$\frac{1}{2}m(t)$	$\frac{1}{2}m'(t)$	$m'(t)$ = response of filter with transfer function $H_Q(f)$ due to message signal $m(t)$. The $H_Q(f)$ is defined by the formula (see part (ii) of footnote 4)
(b) Vestige of upper sideband transmitted	$\frac{1}{2}m(t)$	$-\frac{1}{2}m'(t)$	$H_Q(f) = -j[H(f + f_c) - H(f - f_c)]$ where $H(f)$ is the transfer function of the VSB sideband shaping filter.

3.8 Baseband Representation of Modulated Waves and Band-Pass Filters

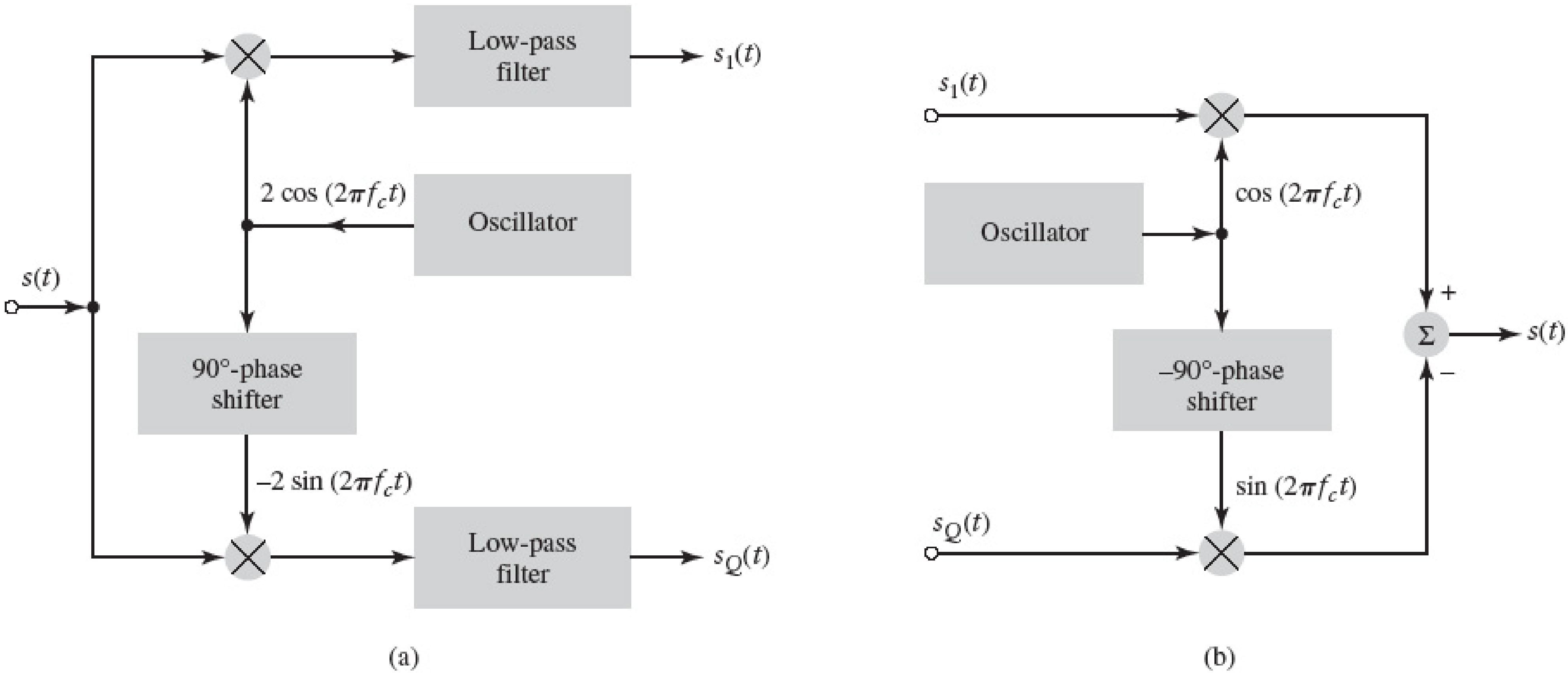


FIGURE 3.25 (*a*) Scheme for deriving the in-phase and quadrature components of a linearly modulated (i.e., band-pass) signal. (*b*) Scheme for reconstructing the modulated signal from its in-phase and quadrature components.

Reminder: 3.5 Quadrature-Carrier Multiplexing

- Quadrature-Amplitude modulation (**QAM**)
  - This scheme enables two DSB-SC modulated waves to occupy the same channel bandwidth (exploiting "quadrature null effect") as one DSB-SC wave

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t) \quad (3.12)$$

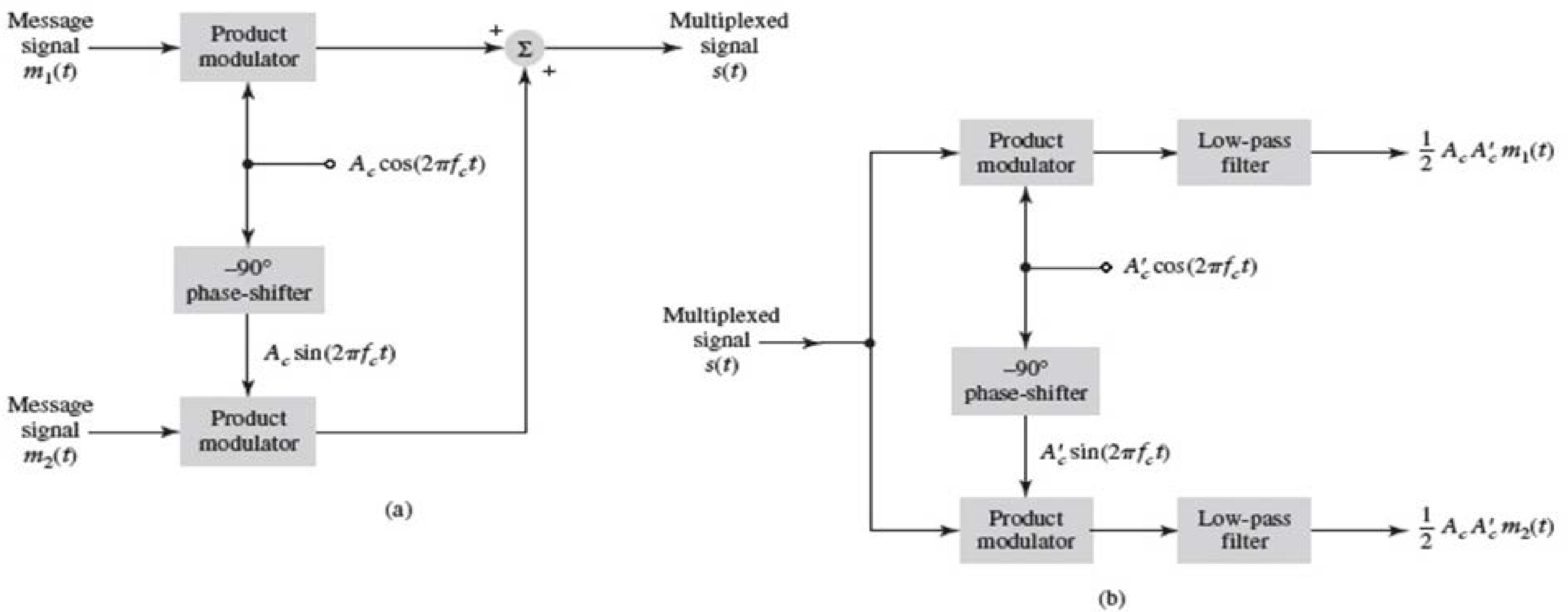


FIGURE 3.17 Quadrature-carrier multiplexing system: (*a*) Transmitter, (*b*) receiver.

❑ Baseband Representation of Band-Pass Filter

Assume the transmission bandwidth of $s(t)$ is $2W$, centered on f_c

Assume the transmission bandwidth of the filter $H(f)$ is $2B$, centered on f_c

consider $B \le W$

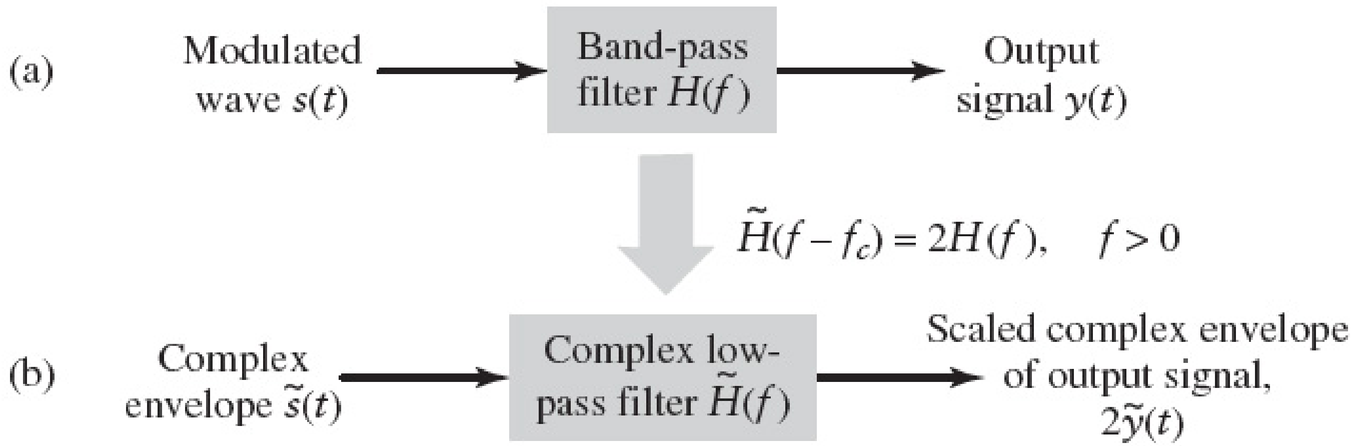


FIGURE 3.26 Band-pass filter to complex low-pass system transformation: (*a*) Real-valued band-pass configuration, and (*b*) corresponding complex-valued low-pass configuration.

3.8 Baseband Representation of Modulated Waves and Band-Pass Filters

- Procedure to determine

  1. Given $H(f)$, defined for both positive and negative frequencies, keep the part of $H(f)$ corresponding to positive frequencies; let $H_+(f)$ denote this part.

  2. Shift $H_+(f)$ to the left along the frequency axis by f_c, and scale it by the factor 2. The obtained result is $\tilde{H}(f)$.

- Actual output $y(t)$ is determined from the formula

$$y(t) = \mathrm{Re}\left[\tilde{y}(t)\exp(j2\pi f_c t)\right] \quad (3.45)$$

3.9 Theme Examples

- The functions of receiver in a broadcasting system
  - Carrier-frequency tuning: select the desired signal
  - Filtering: separate the desired signal from other modulated signals
  - Amplification: compensate for the signal power loss incurred during transmission.
- Superheterodyne Receiver (superhet)

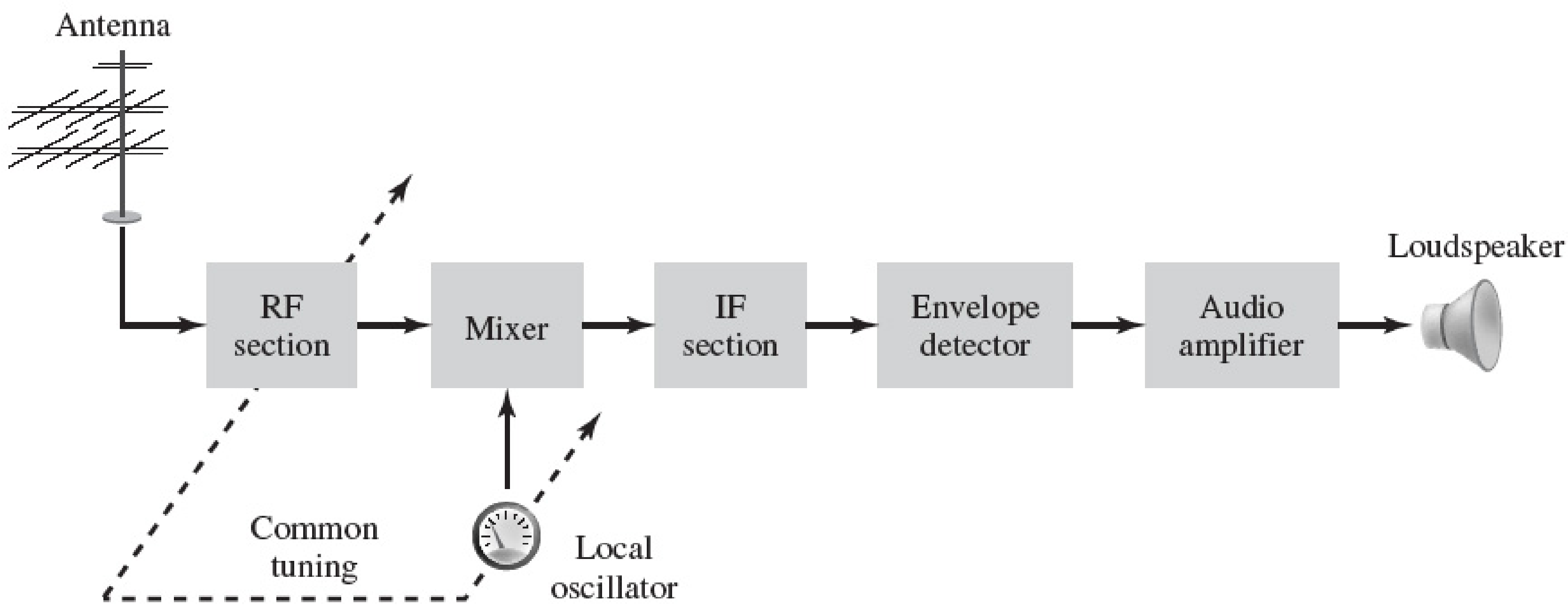


FIGURE 3.27 Basic elements of an AM radio receiver of the superheterodyne type.

3.9 Theme Examples

- Television Signals
  1. The video signal exhibits a large bandwidth and significant low-frequency content, which suggest the use of VSB modulation.
  2. The circuitry used for demodulation in the receiver should be simple and therefore inexpensive. This suggest the use of envelope detection, which requires the addition of a carrier to the VSB modulated wave.

3.9 Theme Examples

TABLE 3.2 *Typical Frequency Parameters of AM and FM Radio Receivers*

	AM Radio	*FM Radio*
RF carrier range	0.535–1.605 MHz	88–108 MHz
Mid-band frequency of IF section	0.455 MHz	10.7 MHz
IF bandwidth	10 kHz	200 kHz

Voice frequency band ranges from approximately 300 Hz to 3400 Hz

The bandwidth allocated for a single voice-frequency transmission channel is usually 4 kHz, including guard bands, allowing a sampling rate of 8 kHz to be used as the basis of the pulse code modulation system used for the digital PSTN.

3.9 The

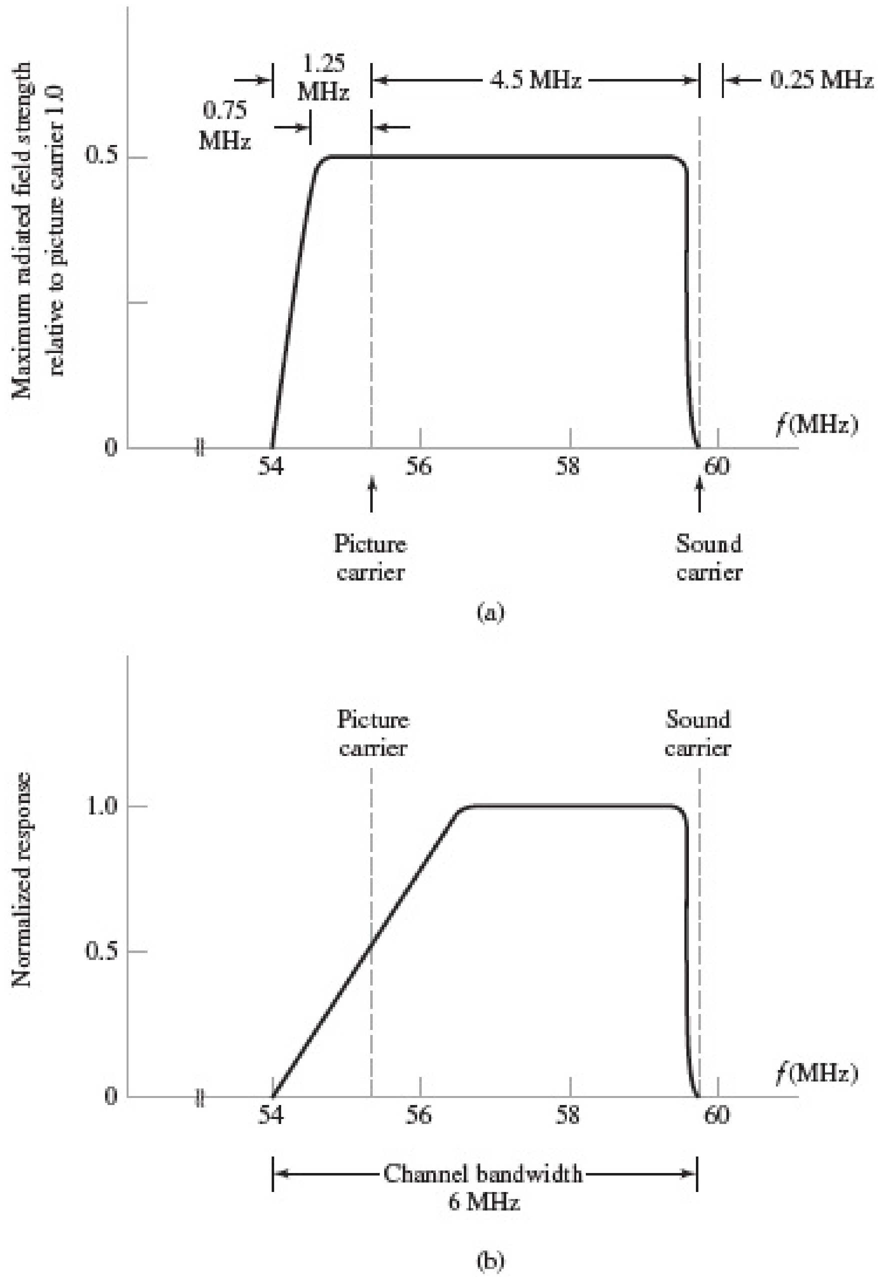


FIGURE 3.28 (*a*) Idealized amplitude spectrum of a transmitted TV signal. (*b*) Amplitude response of a VSB shaping filter in the receiver.

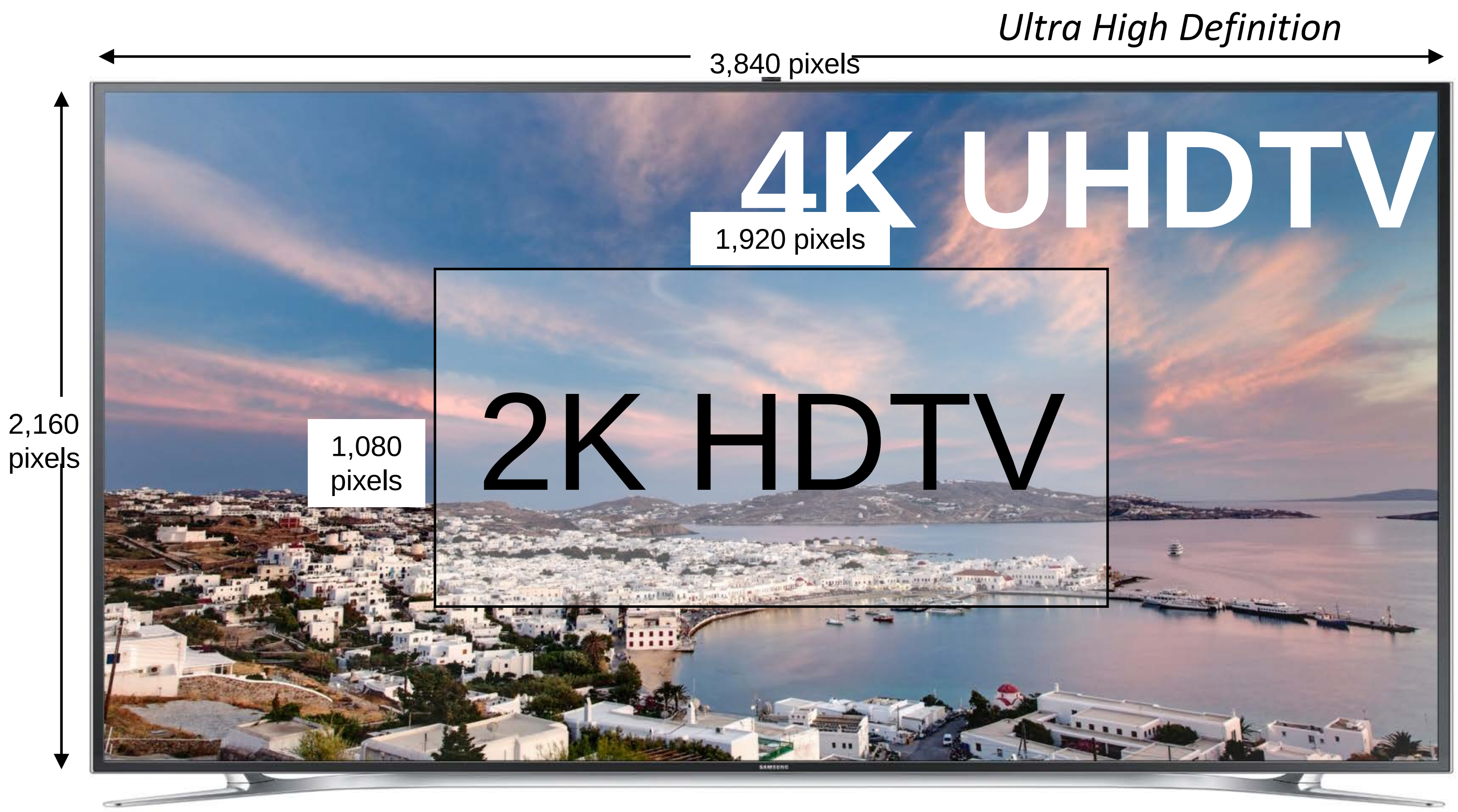
	ATSC 1.0	ATSC 3.0
전송방식	Single Carrier	Multi-Carrier (OFDM, 8K/16K/32K FFT)
TV Network	MFN	MFN, SFN
Channel bandwidth	6 MHz	6, 7, 8 MHz
Target Device	TV	TV, In-door/Mobile Device
Modulation	8-VSB	QPSK, NUC-16/64/256/1K/4K
FEC	RS(207,187) + Conv. (R=2/3)	BCH, CRC + LDPC (R=2/15, 3/15, ... , 13/15)
Data rate (BW = 6MHz)	19.4 Mbps (@SNR 15 dB)	1 Mbps ~ 60 Mbps (SNR: - 5dB ~ 40dB) 25 Mbps (@SNR 15dB)
Multiplexing	None	TDM (time-division multiplexing), LDM (layered division multiplexing)
Time interleaver 길이	4 ms	0, 50, 100, 150, 200 ms
EWS 지원	None	지원

ATSC: Advanced Television Systems Committee

* EWS: Emergency Warning System


4세대
Ultra HD
3세대
디지털 TV
2세대
컬러 TV
1세대
흑백 TV
1956
1980
2001
2013
1981
1991
2000
2012
1세대 아날로그
2세대 GSM/CDMA
3세대 WCDMA/CDMA2000
4세대 LTE

❑ 4K UHD (3840x2160), Digital Cinema (4096x2160), 8K UHD (7680x4320)



3.9 Theme Examples

Multiplexing

- To transmit a number of signals over the same channel, the signals must be kept apart so that they do not interfere with each other, and thus they can be separated at the receiving end.
- Frequency-division multiplexing (FDM)
- Time-division multiplexing (TDM)

Duplexing

- To separate the direction of communications.
- E.g., uplink/downlink
- Frequency-division duplexing (FDD)
- Time-division duplexing (TDD)

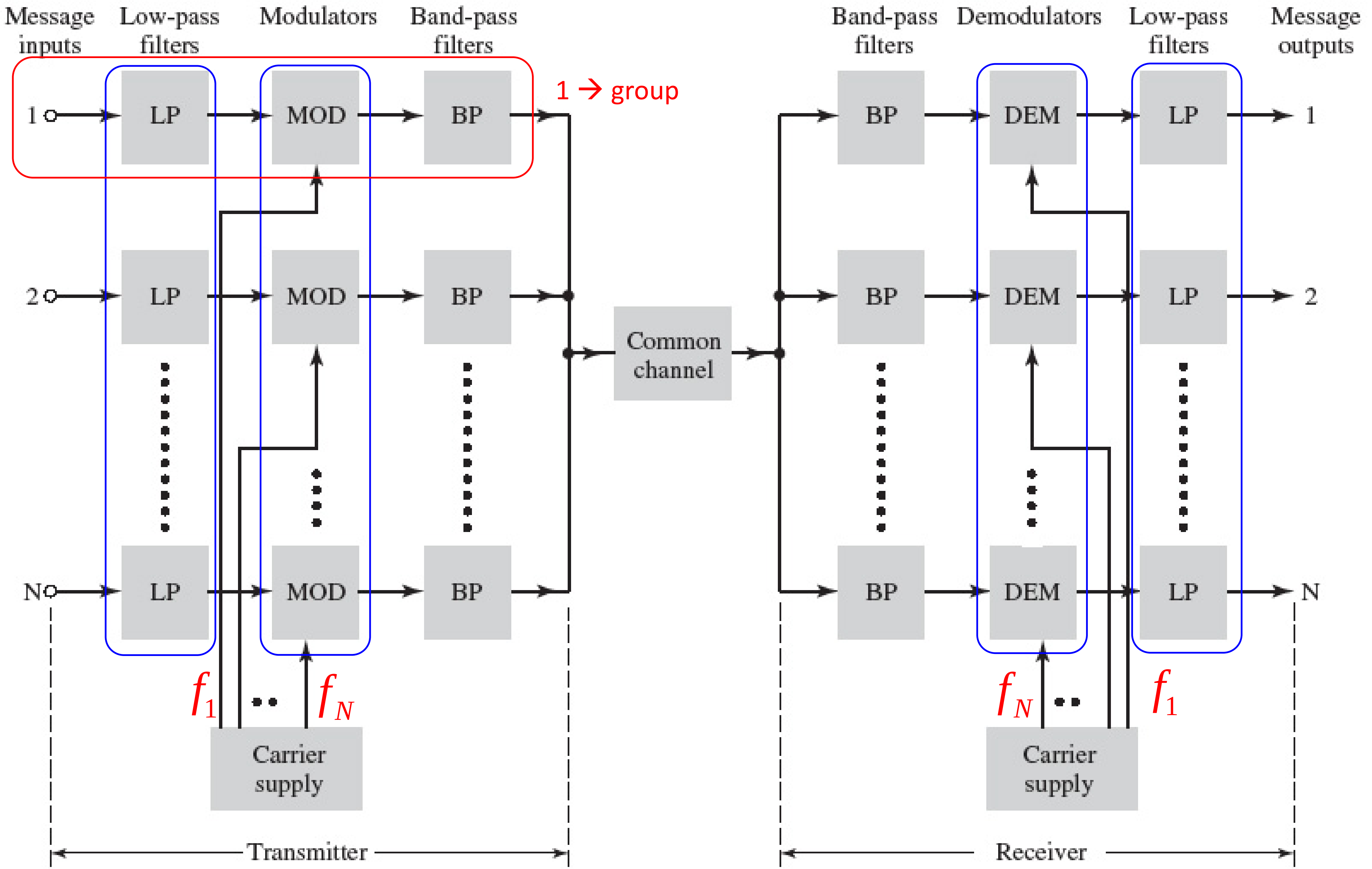


FIGURE 3.29 Block diagram of frequency-division multiplexing (FDM) system.

3.9 Theme Examples

EXAMPLE 3.6 Modulation steps in a 60-channel FDM system

The practical implementation of an FDM system usually involves many steps of modulation and demodulation, as illustrated in Fig. 3.30. The first multiplexing step combines 12 voice inputs into a *basic group*, which is formed by having the nth input modulate a carrier at frequency $f_c = 60 + 4n$ kHz, where $n = 1, 2, \ldots, 12$. The lower sidebands are then selected by band-pass filtering and combined to form a group of 12 lower sidebands (one for each voice input). Thus the basic group occupies the frequency band 60–108 kHz. The next step in the FDM hierarchy involves the combination of five basic groups into a *supergroup*. This is accomplished by using the nth group to modulate a carrier of frequency $f_c = 372 + 48n$ kHz, where $n = 1, 2, \ldots, 5$. Here again the lower sidebands are selected by filtering and then combined to form a supergroup occupying the band 312–552 kHz. Thus, a supergroup is designed to accommodate 60 independent voice inputs. The reason for forming the supergroup in this manner is that economical filters of the required characteristics are available only over a limited frequency range. In a similar manner, supergroups are combined into *mastergroups*, and mastergroups are combined into *very large groups*.

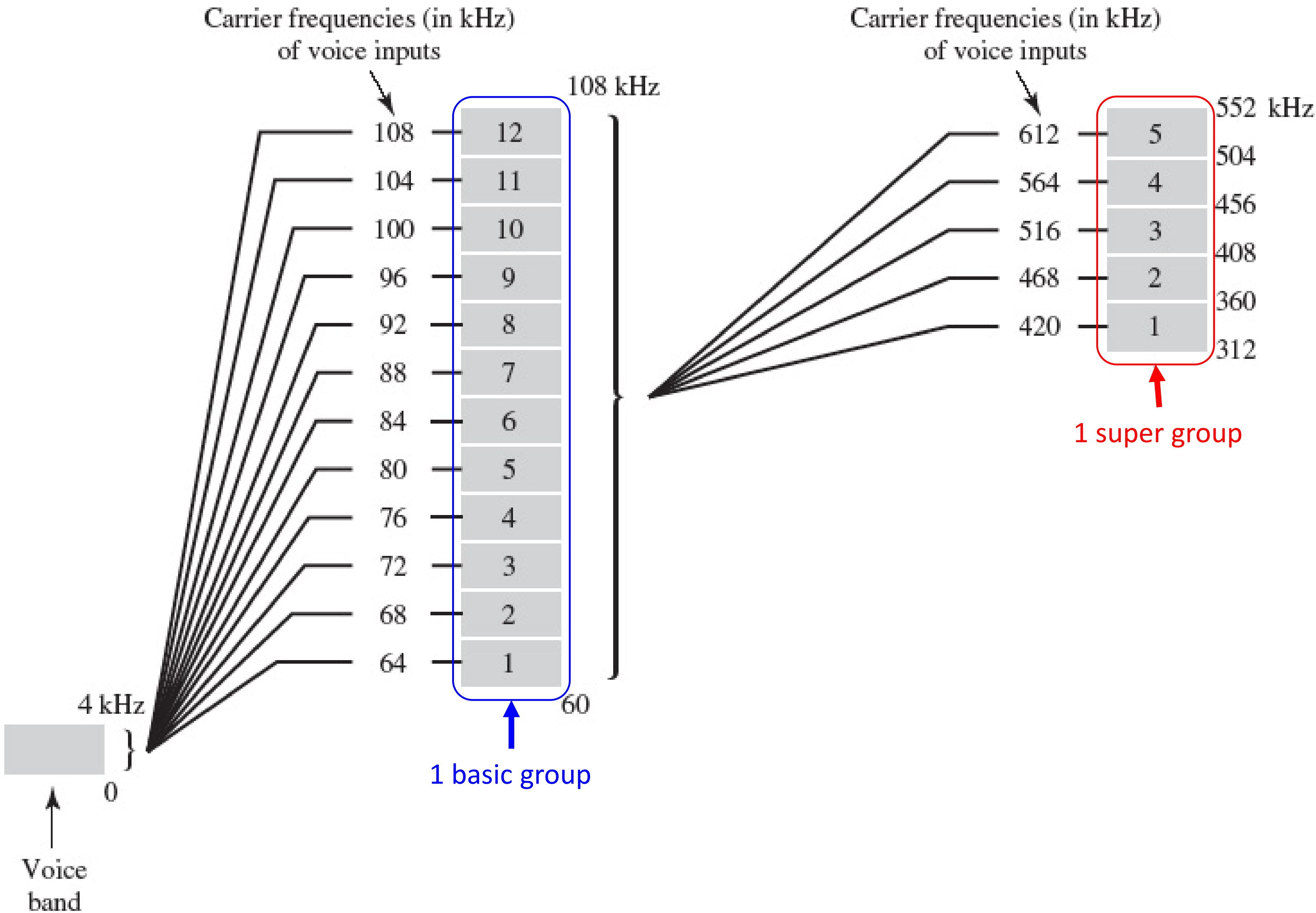


FIGURE 3.30 Illustration of the modulation steps in an FDM system.

3.10 Summary and Discussion

- The example modulated wave is $s(t) = A_c m(t)\cos(2\pi f_c t)$ (3.47)

1. **<u>Amplitude modulation (AM),</u>** in which the upper and lower sidebands are transmitted in full, accompanied by the carrier wave
   - Simple envelope detector vs. wasteful BW and powe (double BW and carrier power) consumption
2. **<u>Double sideband-suppressed carrier (DSB-SC)</u>** modulation, in which only the upper AND lower sidebands are transmitted.
   - Less power than AM vs. Complexity.
3. **<u>Single sideband (SSB) modulation</u>**, in which only the upper sideband OR lower sideband is transmitted.
   - Minimum transmit power and BW vs. Complexity
4. **<u>Vestigial sideband modulation</u>**, in which “almost” the whole of one sideband and a “vestige” of the other sideband are transmitted in a prescribed complementary fashion
   - VSB modulation requires an channel bandwidth that is intermediate between that required for SSB and DSB-SC systems, and the saving in bandwidth can be significant if modulating signals with large bandwidths are being handled.