호남대학교 전자공학과

초고주파 회로 설계 : S-Parameter, Smith-chart

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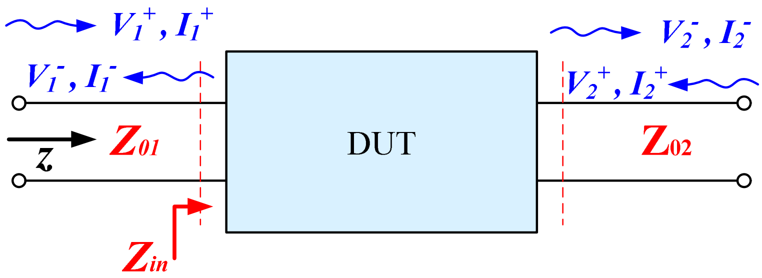
학습 목표

- 회로 특성 파악을 위한 S-parameter 이해
- S-parameter가 갖는 물리적 의미
- 반사계수를 표현한 Smith-chart 이해
- Smith-chart를 통한 임피던스 읽기
- 회로 연결에 따른 Smith-chart에서의 임피던스 이동 경로 파악

2-port Network

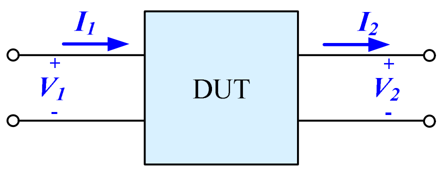


DUT : device under test

V_1^+ : 1port에서의 입사파 전압, I_1^- : 1port에서의 반사파 전류

V_2^- : 2port에서의 반사파 전압, I_2^+ : 2port에서의 입사파 전류

Impedance Matrix



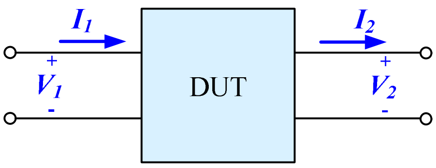
임피던스 파라메터 impedance-parameter

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ Z_{N1} & \cdots & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} \quad \text{or} \quad [V] = [Z][I]$$

where Z_{ij} is found from

$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_k = 0 \text{ for } k \neq j}$$

Admittance Matrix



어드미턴스 파라메터 admittance-parameter

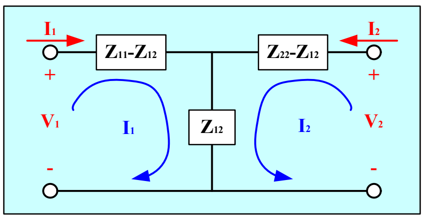
$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ Y_{N1} & \cdots & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad \text{or} \quad [I] = [Y][V]$$

where Z_{ij} is found from

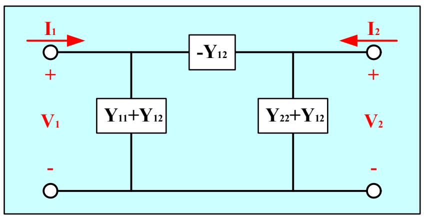
$$Y_{ij} = \left. \frac{I_i}{V_j} \right|_{V_k = 0 \text{ for } k \neq j}$$

2-port 회로망의 등가 회로

T-equivalent circuit for 2-port network



pi-equivalent circuit for 2-port network



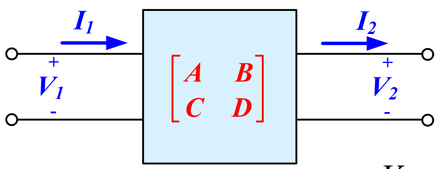
Kirchhoff voltage law

$$V_1 = I_1(Z_{11} - Z_{12}) + I_1 Z_{12} + I_2 Z_{12}$$
$$= Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = I_2(Z_{22} - Z_{12}) + I_2 Z_{12} + I_1 Z_{12}$$
$$= Z_{12} I_1 + Z_{22} I_2$$

Kirchhoff current law

$$I_1 = V_1(Y_{11} + Y_{12}) - (V_1 - V_2) Y_{12}$$
$$= Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = V_2(Y_{22} + Y_{12}) - (V_2 - V_1) Y_{12}$$
$$= Y_{12} V_1 + Y_{22} V_2$$

ABCD parameter



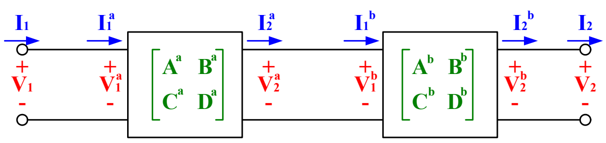
$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

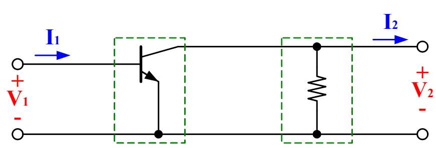
- ABCD 파라메터 ABCD-parameter

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

ABCD-parameter for Cascade Connection

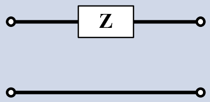
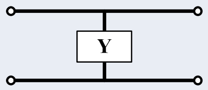
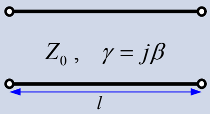
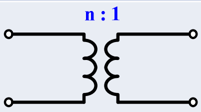
- Cascade connection



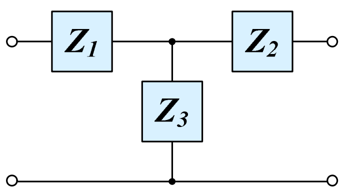


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A^a & B^a \\ C^a & D^a \end{bmatrix} \begin{bmatrix} A^b & B^b \\ C^b & D^b \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Example of ABCD-parameter

2-port Circuit	ABCD-parameter	
	$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$	$V_1 = V_2 + ZI_2$ $I_1 = I_2$
	$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$	$V_1 = V_2$ $I_1 = YV_2 + I_2$
	$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & Z\sin\beta l \\ jY\sin\beta l & \cos\beta l \end{bmatrix}$	$V_1 = V_2 e^{j\beta l} = V_2\cos\beta l + jV_2\sin\beta l$ $= V_2\cos\beta l + jZ_0 I_2\sin\beta l$ $I_1 = I_2 e^{j\beta l} = jZ_0 V_2\sin\beta l + I_2\cos\beta l$
	$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}$	$V_1 = NV_2$ $I_1 = \frac{1}{N} I_2$

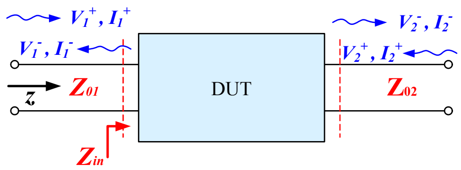
Example of ABCD-parameter



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_3 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{Z_1}{Z_3} & Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} \\ \frac{1}{Z_3} & 1 + \frac{Z_2}{Z_3} \end{bmatrix}$$

Scattering Parameter



$$V(x) = V^+(x) + V^-(x)$$
$$I(x) = I^+(x) - I^-(x) = \frac{V^+(x)}{Z_0} - \frac{V^-(x)}{Z_0}$$

$$v(x) = a(x) + b(x)$$
$$i(x) = a(x) - b(x) \quad , \quad a(x) = \frac{V^+(x)}{\sqrt{Z_0}} \;\&\; b(x) = \frac{V^-(x)}{\sqrt{Z_0}}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ S_{N1} & \cdots & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} \quad \text{or} \quad [b] = [S][a]$$

Scattering Parameter

$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$ 2port가 완전히 termination되어 있을 때의 1port에서의 **입력 반사계수** (reflection coefficient)

$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$ 2port가 완전히 termination되어 있을 때의 1port에서의 **투과계수** (forward transmission coefficient)

$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$ 1port가 완전히 termination되어 있을 때의 2port에서의 **출력 반사계수** (reflection coefficient)

$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$ 1port가 완전히 termination되어 있을 때의 2port에서의 **투과계수** (reverse transmission coefficient)

$$\text{Return Loss} = 10\log|S_{ii}|^2 \quad [\text{dB}]$$
$$\text{Insertion Loss} = 10\log|S_{ij}|^2 \quad [\text{dB}]$$

Scattering Parameter

Reciprocal	[S] matrix is symmetric.
Lossless	[S] matrix is unitary.

- For 2-port network,

$$[S]=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

- For lossless network, it should be $|S_{11}|^2+|S_{21}|^2=1$

- For reciprocal network, it should be $S_{12}=S_{21}$

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Matrix 변환

	S	Z	Y	$ABCD$
S_{11}		$\frac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}		$\frac{2Z_{12}Z_0}{\Delta Z}$	$\frac{-2Y_{12}Y_0}{\Delta Y}$	$\frac{2(AD-BC)}{A+B/Z_0-CZ_0-D}$
S_{21}		$\frac{2Z_{21}Z_0}{\Delta Z}$	$\frac{-2Y_{21}Z_0}{\Delta Y}$	$\frac{2}{A+B/Z_0-CZ_0-D}$
S_{22}		$\frac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$		$\frac{Y_{22}}{\lvert Y\rvert}$	$\frac{A}{C}$
Z_{12}	$Z_0\frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$		$\frac{-Y_{12}}{\lvert Y\rvert}$	$\frac{AD-BC}{C}$
Z_{21}	$Z_0\frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$		$\frac{-Y_{21}}{\lvert Y\rvert}$	$\frac{1}{C}$
Z_{22}	$Z_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$		$\frac{Y_{11}}{\lvert Y\rvert}$	$\frac{D}{C}$
Y_{11}	$Y_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{22}}{\lvert Z\rvert}$		$\frac{D}{B}$
Y_{12}	$Y_0\frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{12}}{\lvert Z\rvert}$		$\frac{BC-AD}{B}$
Y_{21}	$Y_0\frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{21}}{\lvert Z\rvert}$		$\frac{-1}{B}$
Y_{22}	$Y_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{11}}{\lvert Z\rvert}$		$\frac{A}{B}$
A	$\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}}{\lvert Z_{21}\rvert}$	$\frac{-Y_{22}}{Y_{21}}$	
B	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{\lvert Z\rvert}{Z_{12}}$	$\frac{-1}{Y_{21}}$	
C	$\frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{1}{Z_{21}}$	$\frac{-\lvert Y\rvert}{Y_{21}}$	
D	$\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	

스미스 차트 (Smith Chart)

- 1939년에 Phillip Smith에 의해 고안.
- 복소수 평면에서 반사계수 표현

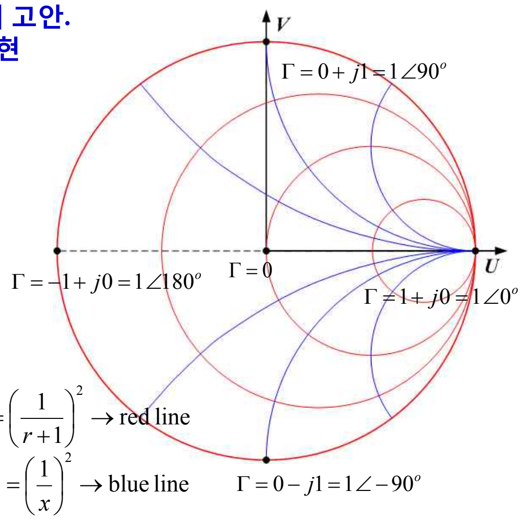
$$\Gamma = \frac{z_L - 1}{z_L + 1} \quad \leftarrow \quad z_L = r + jx$$

$$= \frac{(r-1) + jx}{(r+1) + jx} = U + jV$$

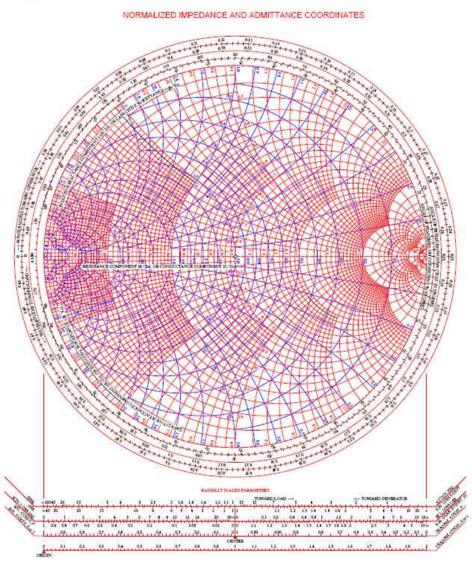
$$U = \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} \qquad \left(U - \frac{r}{r+1}\right)^2 + V^2 = \left(\frac{1}{r+1}\right)^2 \rightarrow \text{red line}$$

$$\Rightarrow$$

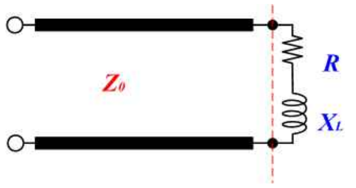
$$V = \frac{2x}{(r+1)^2 + x^2} \qquad (U-1)^2 + \left(V - \frac{1}{2}\right)^2 = \left(\frac{1}{x}\right)^2 \rightarrow \text{blue line}$$



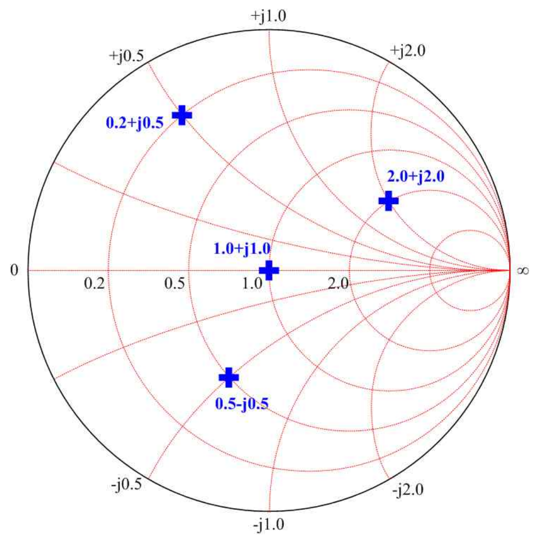
스미스 차트 (Smith Chart)



반사 계수의 표현



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

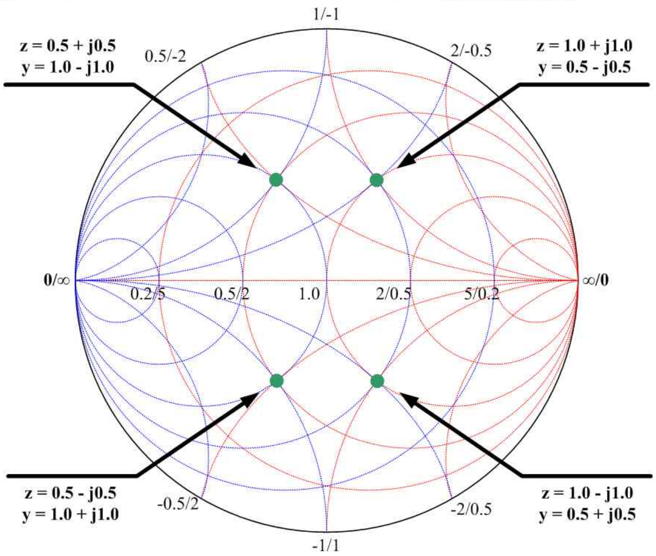


임피던스 & 어드미턴스 차트

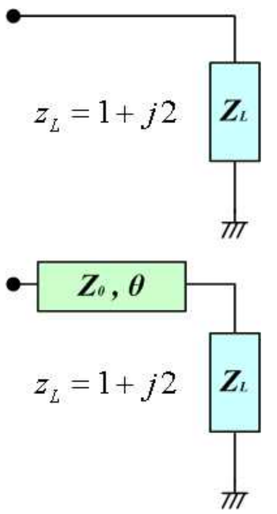
$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - z_0}{z_L + z_0} = \frac{y_0 - y_L}{y_0 + y_L}$$

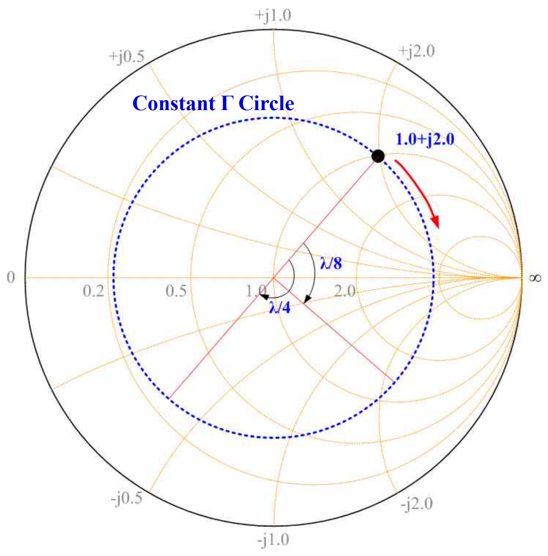
$$z = Z/Z_0$$

$$y = Y/Y_0$$

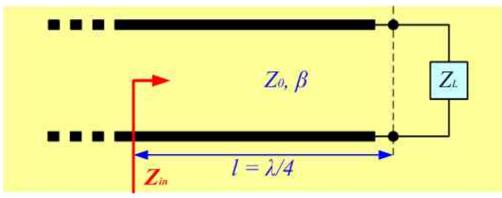


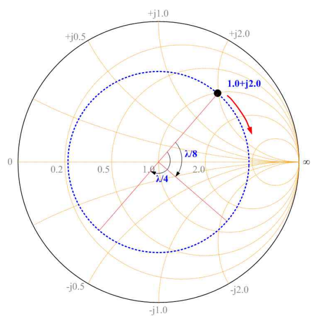
전송선로 연결에 따른 반사 계수 변화





λ/4 전송선로 연결



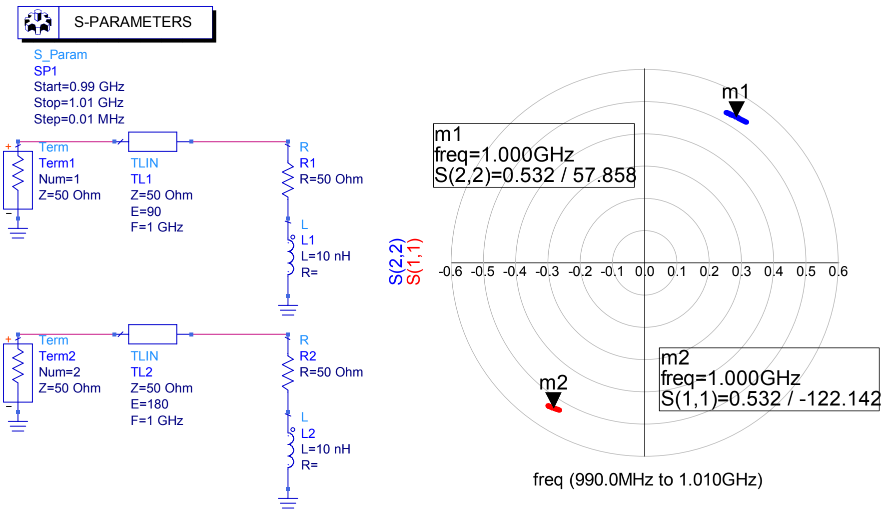


$$Z_{in} = Z_0 \left. \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right|_{l=\lambda/4} = \frac{Z_0^2}{Z_L}$$

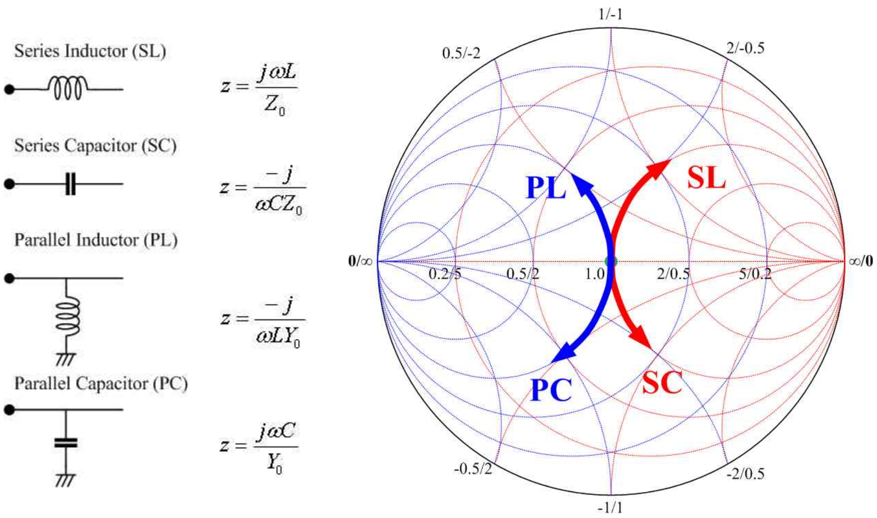
$$z_{in}\Big|_{l=\lambda/4} = \frac{1}{z_L} = y_L$$

open circuit $\underset{\lambda/4}{\leftrightarrow}$ short circuit

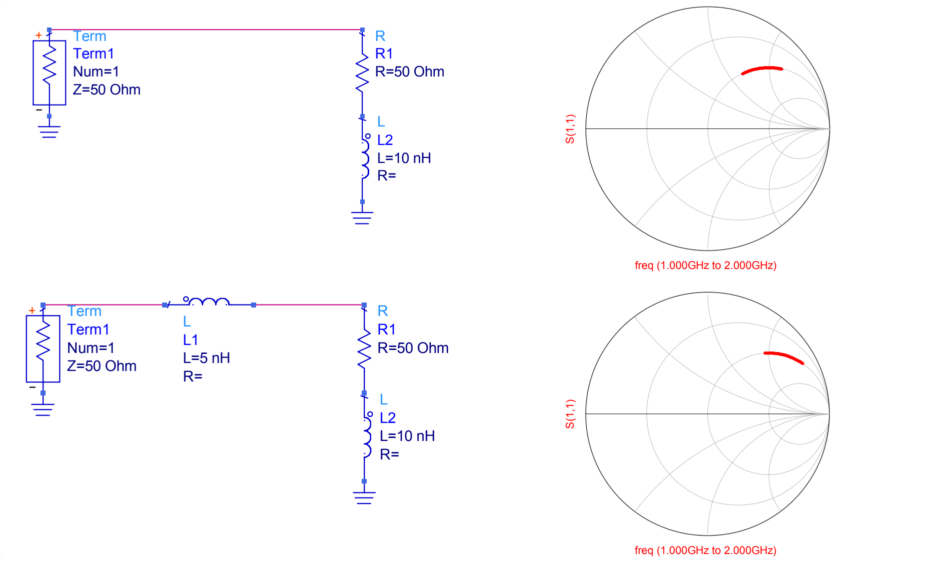
Example : Connection of Transmission-line



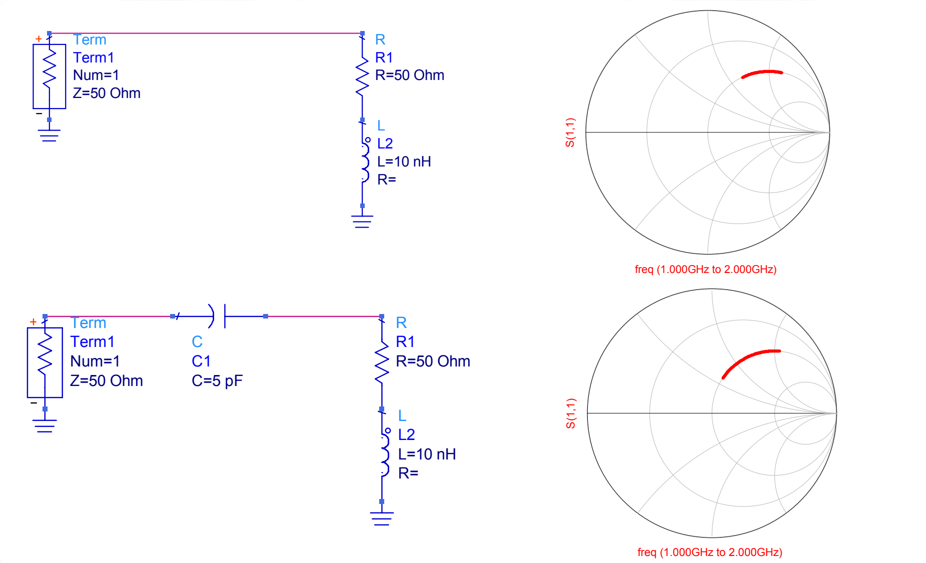
LC 연결에 따른 반사 계수 변화



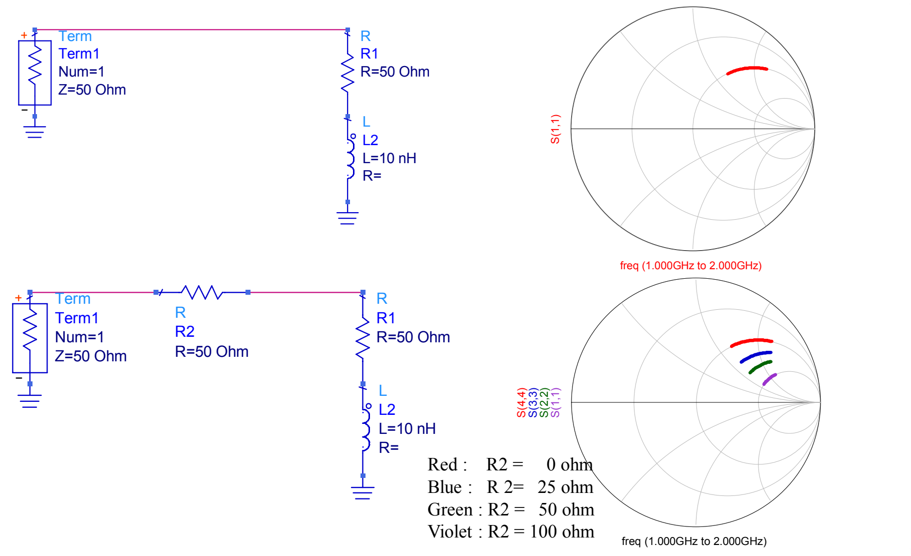
Example : Connection of Series L



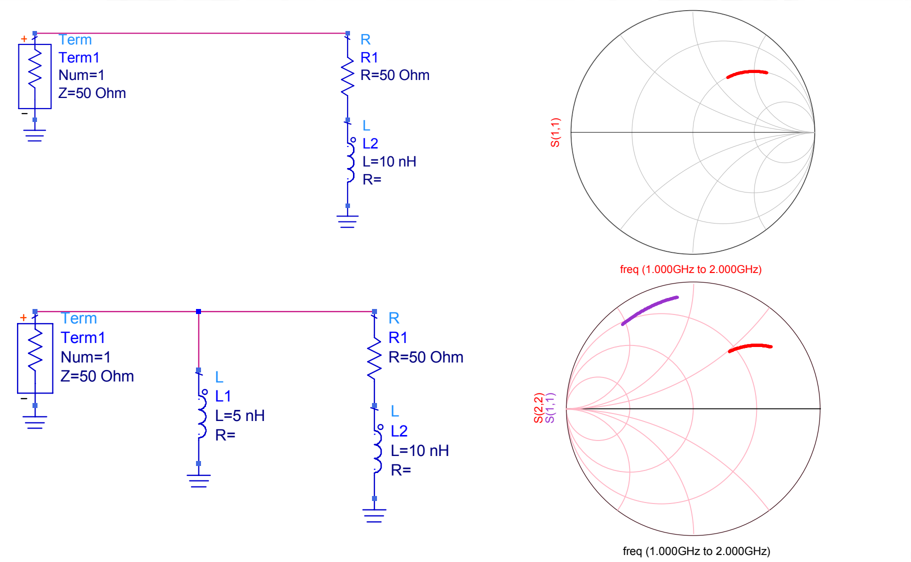
Example : Connection of Series C



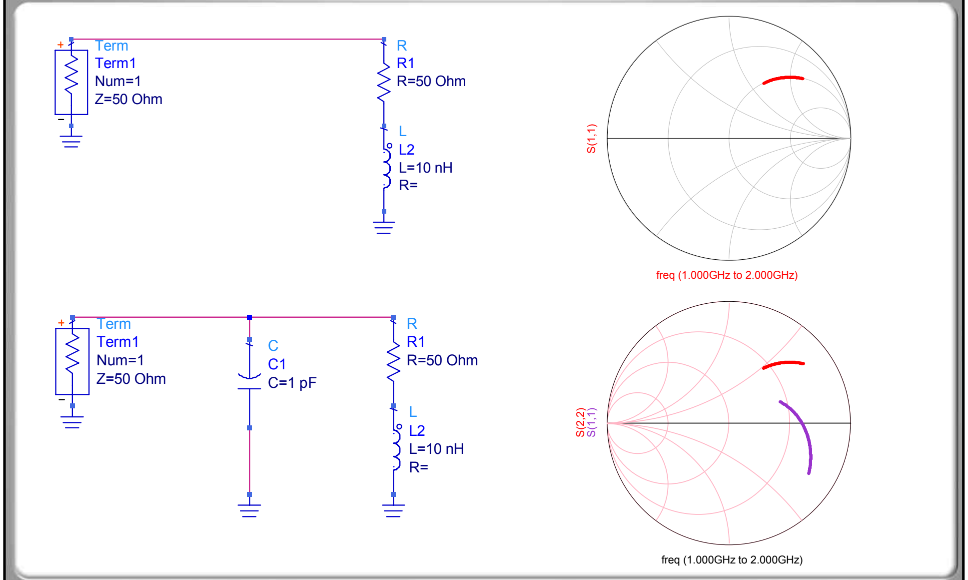
Example : Connection of Series R



Example : Connection of Parallel L



Example : Connection of Parallel C



Example : Connection of Parallel R